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Process Capability Studies – The Better Way

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Abstract
To estimate the ability of a process to perform according to the product's design (i.e., specification), process capability indices (C_p, C_{pk}, etc.) are used. What most people fail to realize, however, is that the actual process capability indices should be based on population parameters (e.g., mean and variance), but we rarely, if ever, have that information. In fact, it is almost always the case that *samples* from the population are chosen to *estimate* the process's parameters and, therefore, the process capability indices that are typically reported are actually process capability estimates.

This paper will introduce (to the new reader) or refresh (to the learned reader) the basic concepts and formulas for process capability estimation. Subsequent to this, formulas for process capability confidence intervals (CIs) will be provided. An experiment is presented to highlight the effect of sample size on the process capability estimates and CIs.

This paper does not intend to introduce new formulas for process capability CI generation. In fact, these formulas have been around for quite a number of years. Surprisingly, however, most of the applied work in electronics manufacturing articles that is mentioned with regard to process capability fail to use these formulas. The purpose of this paper is not to highlight the many practitioners and researchers that have failed to mention process capability confidence intervals in their work (we also would be included in that list), but it serves to bring to a wider audience (in particular, the electronics manufacturing audience) the suggestion to utilize these formulas so that they and their customers (or suppliers) can get a broader picture of process capability.

Keywords: *Process Capability, Confidence Intervals, Electronics Manufacturing*

Introduction

Far too often, we find process capability articles in the literature that report results on a process with the use of some process capability point-estimate index. Many of us know that a simple process capability index (like C_p, C_{pk}, etc.) is calculated as a ratio of the tolerance of some feature (print tolerance, X-axis accuracy, placement force, etc.) to the variability of the process (as typically measured as a function of standard deviation). However, what many of us may not realize is that this process capability index, while taken for granted, is simply a point-estimator (e.g., $\hat{C}_{pk}$) for the process performance (C_{pk}) and is based upon a *sample* of data. In the calculation of the index, it is typical to take 30 or more readings from the process, and then calculate the metric. Taking such a sample from the process is just that – a sample. As an example, three different samples, all of size 50, may show an average process (C_{pk} = 1.00), a good one (C_{pk} = 1.47) or a bad one (C_{pk} = 0.7), based on the samples being used, but the true C_{pk} may be none of these values! (Dogdu, 1999). Any of the indices, however, is a legitimate estimate of the process capability. Why are they different? Because they are from different samples!

Any given process will have a range or an interval of process capability estimates. Within that interval lies a true process capability (subject to some level of confidence, α). This means that when a point estimate of the process capability is used, the practitioners are merely picking a single estimate out of that range filled with other candidates. Therefore, reporting a single point estimate of the process capability, while leaving out the rest of the estimates, hides information about the true capability of the process. The purpose of this paper is to introduce the concepts of confidence interval usage with process capability indices. The use of confidence intervals will provide you and your customers a better understanding of where the actual process capability lies, by reporting an interval that is likely to contain the true capability, as opposed to just simply providing a single point estimate that probably is not, exactly, the true capability.

Process Capability Estimates

Traditional process capability formulas can be found in many textbooks (see Kolarik (1999), or Gryna (2001), for example). Furthermore, many of these indices are often covered in specialized short courses for Statistical Process Control (SPC) study (their confidence intervals tend not to be covered in short courses!). Nonetheless, they will briefly be covered (or reviewed) before the process capability confidence intervals are introduced.

The process capability indices that will now be reviewed are C_p, C_{pk}, and C_{pm} as they are the most utilized (for other metrics, the interested reader may refer to any newer textbook in the area of SPC).

"Inherent" Process Capability, C_p, Index

One of the commonly used indices is the C_p index, sometimes referred to as the "inherent" process capability index. The index is a measure used to compare the natural process variation and the tolerance limits, namely, the difference between the Upper Specification Limit (USL) and the Lower Specification Limit (LSL). The C_p index is expressed as the following ratio:

$$C_p = \frac{USL - LSL}{6\sigma}$$

The numerator is the tolerance spread and the denominator represents the process spread. Indices greater than one indicate a process that is "potentially" capable of meeting the specifications; while an index less than one indicates that the process will tend to fail to meet the specification (i.e., will have poor yields). In short, the higher the C_p index, the better the potential of the process to meet the specifications and have higher yields.

One of the shortcomings of the C_p index is that it fails to account for the centering of the process. It is conceivable that a process that is centered outside the specification limits can yield a high C_p index. Clearly, such a process is going to yield a lot of defects, yet the index fails to capture that reality (an example of this will be presented below).

"Actual" Process Capability, C_{pk}, Index

Although C_p provides information about the capability of a process, by definition it is the ratio of the specification limits to the process variation and does not take into account the centering of the process. C_{pk}, on the other hand, takes into account the centering of the process as well as the specification limits and is defined as:

$$C_{pk} = \min\left\{\frac{\mu - LSL}{3\sigma}, \frac{USL - \mu}{3\sigma}\right\}$$

If we focus on the case of strictly looking at process capability by only considering the process variation (i.e., restricting ourselves to the inherent process capability), we can run into a major pitfall. Figure 1 demonstrates a case where C_p is 1 although the process is clearly off-center. In this case, C_{pk} will have a negative value and indicates that the process is not "actually" capable.

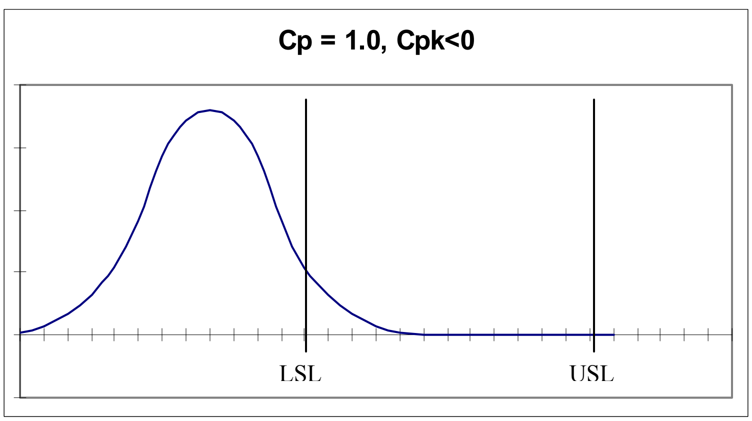

Figure 1 - C_p & C_{pk} Example

C_{pm} Index

As C_{pk} was devised to make up for the major shortcoming of C_p (the failure to incorporate the location (centering) of the process), other researchers have proposed another process capability index, namely, C_{pm}, to address a seeming related, but different, shortcoming. While C_{pk} reflects the process as being nearer to either the USL or LSL, neither C_p nor C_{pk} address the degree to which the process is away from the target (Chan, Cheng, and Spiring, 1998). Consequently, the C_{pm} index, below, is devised to capture this information.

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

It is easy to recognize that when the centering of the process, μ, is located at the nominal (target) value, T, then C_{pm} simplifies, as it should, to the C_p index.

The reader well-versed in quality assurance concepts may recognize that this shortcoming is related to Taguchi's quality loss concept (see Kolarik, 1999, for reference). One of Taguchi's major points is that, while a product may be within specifications, if the product is not at the target (nominal) value, then there is a loss to society from this product and that the loss is greater the further we are removed from the nominal value. This C_{pm} formula works in a related fashion. The further the process centering is away from the target value, the worse the C_{pm} result will be (because of the $(\mu - T)^2$ term in the denominator).

Sample Statistics

What it all boils down to is the following – this work reflects the fact that people are trying to glean information about a population of data points as taken from a sample. When taking sample data from a population, it is important to keep in mind that the data points should be obtained randomly. As a bad example, but unfortunately a realistic one, consider the case in that all of the data points in the process capability study were taken around one point of time in the day (and the process is known to vary (with assignable causes) throughout the day). Therefore, the process capability estimate taken from that sample is not really an estimate of the overall process, it is more of an estimate of the process capability at that time of the day and, in fact, that estimate really should not, in our opinions, be utilized. This actually brings up another important point.

The process should be stable (i.e., as verified through SPC control charts) before determining process capability estimates. By being stable, all assignable causes of variation should be eliminated. Once the process is shown to be stable, this helps ensure that the sample data taken from the process are not subject to special conditions (e.g., taken during the best times) and help to reflect the overall process capability (albeit an estimate). Furthermore, tests of normality (that can be done via the help of spreadsheets or more tailored statistical packages (such as Minitab)) should be performed as the formulas (both above and below) assume normality of the data.

Process Capability Confidence Intervals

Since it is rarely the case that we have population parameters, we are not actually calculating the capability of the process (regardless of metric that we are using). The metrics that we are using (as based upon sample mean, sample standard deviation, etc.) are process capability estimates. As such, they should be appropriately treated (and named). By utilizing the sample data parameters, the aforementioned process capability formulas should be modified as below.

$$\hat{C}_p = \frac{USL - LSL}{6s}$$

$$\hat{C}_{pk} = \min\left\{\frac{\overline{X} - LSL}{3s}, \frac{USL - \overline{X}}{3s}\right\}$$

In the above formulas, the "^" symbol represents that the value is an estimate. Furthermore, we replace the population parameters for mean and standard deviation (μ and σ), with their sample data estimates ($\overline{X}$ and s).

Inherent Process Capability (C_p) Confidence Interval

In order to develop a confidence interval, we have to state a level of significance (α) for the calculation. Alpha usually varies from 1% to 10%, but it is typically chosen as 5%. Since inherent process capability only uses one parameter (σ), a 100(1-α)% CI for C_p can be directly based upon the 100(1- α)% CI for σ that can be found in any basic statistics text, with the resulting C_p CI below.

$$\hat{C}_p\sqrt{\frac{\chi^2_{1-\alpha/2,n-1}}{n-1}} \le C_p \le \hat{C}_p\sqrt{\frac{\chi^2_{\alpha/2,n-1}}{n-1}}$$

Cp CI Example

Assume we have a part whose USL is 120mil and the LSL is 100mil. Furthermore, let us assume that 30 of these parts picked randomly from a process have yielded a sample standard deviation, s, of 2.35. The calculations below show how to generate a 95% CI (a = 5%) starting first with the estimate of the inherent process capability.

$$\hat{C}_p = \frac{120-100}{6(2.35)} = \frac{20.0}{14.1} = 1.42$$

This is the point where many practitioners, and some researchers, would stop with a statement that the inherent process capability is 1.42. But 1.42 is not the inherent process capability, it is only an estimate of the inherent process capability.

To continue, we would now need to determine the chi-squared values (from statistical tables). For our example, the relevant chi-squared values are below.

$$\chi^2_{0.975,29} = 16.047 \qquad \chi^2_{0.025,29} = 45.722$$

Upon determination of the chi-squared values, and already knowing our sample size and inherent Cp estimate, we can now use the aforementioned Cp CI formula, below.

$$1.42\sqrt{\frac{16.047}{29}} \le C_p \le 1.42\sqrt{\frac{45.722}{29}}$$

Simplifying, we get the following:
The interpretation is that, with 95% confidence, we can say that the inherent process capability is as small as 1.06 and as large as 1.78. This provides us with more information than just the simple estimate of the inherent process capability.

Confidence Intervals for other Process Capability Metrics
As it has been demonstrated in the previous section, calculating a confidence interval for C_p is straightforward. What makes it easy is because that index is based on a single parameter or statistic, namely the standard deviation *s,* which is χ^2 distributed. Since the numerator, when calculating the C_p, is constant, it then follows that the C_p index will also be χ^2 distributed. Knowing the distribution of the $\hat{C}_p$ indices, one can then obtain the standard error and construct confidence intervals for $\hat{C}_p$ as performed in the previous section. With regard to the one sided indices, such as the C_{pk} and C_{pm}, establishing their underlying distribution of the indices is a bit challenging and controversial. This is largely due to the fact that both indices are based upon two parameters, the average, $\overline{X}$, and the standard deviation, *s*, which are independently distributed and are characterized by the *student t* and χ^2 distributions, respectively. Trying to ascertain the distribution of $\hat{C}_{pk}$ and $\hat{C}_{pm}$ indices and, subsequently, their interval estimates, therefore becomes a challenging task.

Due to the aforementioned reasons, attempts to construct confidence intervals for process capability metrics remain an issue of controversy and academic debate. Most of the controversy centers on the difficulty to execute the proposed methods and also the excessive widths of the intervals (Hoffman, 2001). Tang et al. (1997) for instance, criticizes the method by Kushler and Hurley (1992) for being computationally complex. The method proposed by Franklin and Wasserman (1991) is also computationally complex.

Our paper presents a method to construct interval estimates for the $\hat{C}_{pk}$ index as proposed by Bissell (1990). This method is a compromise in a sense that it is not as computationally complex as many proposed methods yet it is still problematic because it produces wide confidence intervals. For a method to construct interval estimates for the Taguchi index, $\hat{C}_{pm}$, see Zimmer et al. (2001). The following is a formula developed by Bissell (1990) for calculating the CI for the $\hat{C}_{pk}$ index:

$$\hat{C}_{pk}\left(1 - z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^{\ 2}} + \frac{1}{2n-1}}\right) \le C_{pk} \le \hat{C}_{pk}\left(1 + z_{\alpha/2}\sqrt{\frac{1}{9n\hat{C}_{pk}^{\ 2}} + \frac{1}{2n-1}}\right)$$

While there are other methods to generate CIs for the other metrics as stated above, it has been said that the use of the above formula can also apply to the other metrics by simply swapping out the C_{pk} (and its estimator) with another index (and its estimator).

C_{pk} CI Example

Continuing with the same example in 3.1.1, let us assume that the process yielded a sample average, $\overline{X}$, of 110mil. The calculations, below, show how to generate a 95% CI (α = 5%) starting first with the estimate of the process capability.

$$\hat{C}_{pk} = \min\left\{\frac{110-100}{3*2.35}, \frac{120-110}{3*2.35}\right\}$$

$$\hat{C}_{pk} = 1.42$$

$$1.42\left(1-1.96\sqrt{\frac{1}{9*30*1.42^2}+\frac{1}{2*30-1}}\right) \le C_{pk} \le 1.42\left(1+1.96\sqrt{\frac{1}{9*30*1.42^2}+\frac{1}{2*30-1}}\right)$$

For the 95% CI, $z_{\alpha/2}$ or $z_{0.025}$ is 1.96 therefore the CI is: $1.034 \le C_{pk} \le 1.802$

Similar to the earlier example, the interpretation is that, with 95% confidence, we can say that the actual process capability is as small as 1.034 and as large as 1.802.

Experiment

Using the aforementioned method by Bissell (1990), we conducted a simulation experiment to test the validity of the method to estimate the true C_{pk}. We generated a population of normally distributed random numbers with the mean, μ = 7.00 and the population standard deviation, σ = 0.04. We set fictitious USL = 7.2 and LCL = 6.8. From that, we calculated the population C_{pk} = 1.658. The idea is to use the Bissell method to estimate the population C_{pk} using CI and to see how effective the method is in estimating the index.

We randomly sampled, from the population of 10000 normally distributed random numbers, 30 samples of size 15, 30 and 60. We then calculated the CI using the above-mentioned method.

Based on the three experiments (with results depicted in Table 1 of the Appendix), one can see that the confidence intervals manage to include the true or population capability (which is 1.658) except for the cases indicated by the * symbol which seem to occur frequently when the sample size is small (in this case, the sample size of 15). Three observations can be drawn from this experiment. First, the sample size does not affect the width of the CI. All sample sizes produce a wide CI, as indicated by the average length of the intervals. The second observation is that the sample size seems to affect the accuracy of the intervals. The smaller the sample, the more likely are the intervals to exclude the population index. Intervals for the sample size of 15 excluded the population index 3 times (i.e., in trial numbers 8, 28 and 29). The last observation is the presence of a bias; the lower limit is always close to the population index we are trying to estimate.

Conclusions

Process capability indices are useful for identifying the capability of a process to produce parts according to their specifications. Process capability formulas are widely used in industry. However, many practitioners restrict attention to estimates of process capability and fail to provide confidence intervals for the indices. It is our belief that the reason for not providing the CIs is due to lack of knowledge of their existence. As such, this relates to the purpose of this work (which is to spread this knowledge of CI generation of process capability indices to the electronics manufacturing community). As presented in this paper, using process capability indices as a point estimator may provide misleading information, thus ill-informed business decisions may be made. The use of confidence intervals to determine process capability will yield a better way to understand the capability of a process.

References

1. A.F. Bissell, "How Reliable is Your Capability Index?", *Applied Statistics*, Vol. 39, No. 3 (1990), 331-340.
2. L.K. Chan, S.W. Cheng, and F.A. Spiring, "A New Measure of Process Capability, Cpm," Journal of Quality Technology, Vol. 20, 160-175.
3. S. Dogdu, "Measurement and Use of Process Capability in Statistical Process Control Activities in a Printed Circuit Board Manufacturing Domain", *Ph.D. Dissertation*, Binghamton University, Binghamton, NY, 1999.
4. L.A. Franklin, G. Wasserman, "Bootstrap Confidence intervals of Cpk: An Introduction", *Communications in Statistics – Simulation and Computation*, Vol. 20, (1991), 231-242.
5. F.M. Gryna, *Quality Planning and Analysis, 4th Ed.*, McGraw-Hill Irwin, 2001.

6. L.L. Hoffman, "Obtaining Confidence Intervals for C_{pk} Using Percentiles of the Distribution of $\hat{C}_p$", *Quality and Reliability Engineering Int*., Vol. 17, (2001), 113-118.
7. V.E. Kane, "Process Capability Indices", *Journal of Quality Technology*, Vol. 18, No. 1, (1986), 41-52.
8. W.J. Kolarik, Creating Quality: Process Design for Results, WCB/McGraw-Hill, 1999.
9. R. Kushler, P. Hurley, "Confidence Bounds for Capability Indices", *Journal or Quality Technology*, Vol. 24, (1992), 188-195.
10. N. Msimang, "Process Capability Study: New Approaches", *M.S. Thesis*, Binghamton University, Binghamton, NY, 1999.
11. C.T. Tang, S.E. Than, B.W. Ang, "A graphical Approach to Obtaining Limits of C_{pk}", *Quality and Reliability Engineering Int.*, Vol. 13, (1997), 337-346.
12. F.K. Wang, "Confidence Interval for the Mean of Non Normal Data", *Quality and Reliability Engineering Int.*, Vol. 17, (2001), 257-267.
13. L.S. Zimmer, N.F. Hubele, W.J. Zimmer, "Confidence Intervals and Sample Size Determination for C_{pm}", *Quality and Reliability Engineering Int*., Vol. 17, (2001), 51-68.

Appendix

Table 1 - The Results of the Interval Estimation Simulation

	Sample Size = 15			*Sample Size = 30*			*Sample Size = 60*		
	Lower Bound	Upper Bound	Interval Length	Lower Bound	Upper Bound	Interval Length	Lower Bound	Upper Bound	Interval Length
1	1.018	3.335	2.3	1.104	3.180	2.1	1.258	3.874	2.6
2	0.808	2.400	1.6	1.106	3.186	2.1	1.317	4.155	2.8
3	1.161	4.055	2.9	1.146	3.363	2.2	1.147	3.364	2.2
4	1.172	4.111	2.9	1.013	2.797	1.8	1.109	3.202	2.1
5	0.828	2.484	1.7	1.442	4.787	3.3	1.192	3.567	2.4
6	0.811	2.415	1.6	1.092	3.126	2.0	1.187	3.543	2.4
7	1.153	4.012	2.9	1.287	4.009	2.7	1.228	3.732	2.5
8	1.694*	7.311*	5.6	1.104	3.177	2.1	1.226	3.724	2.5
9	1.485	5.923	4.4	1.083	3.088	2.0	1.027	2.857	1.8
10	0.902	2.800	1.9	1.425	4.700	3.3	1.228	3.732	2.5
11	0.801	2.371	1.6	1.289	4.021	2.7	1.344	4.289	2.9
12	1.079	3.634	2.6	1.129	3.286	2.2	1.198	3.593	2.4
13	0.957	3.050	2.1	1.286	4.008	2.7	1.021	2.831	1.8
14	1.100	3.739	2.6	1.238	3.777	2.5	1.401	4.574	3.2
15	1.035	3.418	2.4	1.009	2.783	1.8	1.220	3.694	2.5
16	1.209	4.311	3.1	1.271	3.932	2.7	1.077	3.064	2.0
17	0.967	3.095	2.1	1.089	3.113	2.0	1.219	3.693	2.5
18	0.945	2.995	2.1	1.205	3.625	2.4	1.145	3.355	2.2
19	0.950	3.016	2.1	1.187	3.544	2.4	1.235	3.766	2.5
20	1.123	3.856	2.7	1.547	5.349	3.8	1.364	4.389	3.0
21	1.044	3.463	2.4	1.043	2.919	1.9	1.120	3.246	2.1
22	0.752	2.176	1.4	1.237	3.773	2.5	1.130	3.292	2.2
23	0.819	2.449	1.6	1.049	2.946	1.9	1.185	3.536	2.4
24	0.753	2.181	1.4	1.043	2.923	1.9	1.086	3.101	2.0
25	1.361	5.171	3.8	1.132	3.300	2.2	1.111	3.208	2.1
26	0.985	3.180	2.2	1.179	3.508	2.3	1.245	3.810	2.6
27	0.752	2.178	1.4	1.136	3.318	2.2	1.048	2.940	1.9
28	1.964*	9.305*	7.3	1.067	3.021	2.0	1.372	4.430	3.1
29	1.749*	7.698*	5.9	1.034	2.882	1.8	1.436	4.754	3.3
30	0.912	2.847	1.9	1.100	3.162	2.1	1.222	3.705	2.5
		Average	2.7		Average	2.3	Average		2.4