Robust Reliability Testing For Drop-on-Demand Jet Printing

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Abstract
In this study, the question was how to perform statistically reliable robust- ness tests for the non-contact drop-on-demand printing of functional fluids, such as solder paste and conductive adhesives. The goal of this study was to develop a general method for hypothesis testing when robustness tests are performed. The main problem was to determine if there was a statistical difference between two means or proportions of jet printing devices. In this study, an example of jetting quality variation was used when comparing two jet printing ejector types that differ slightly in design. We wanted to understand if the difference in ejector design can impact jetting quality by performing robustness tests. and thus answer the question, "Can jetting differences be seen between ejector design 1 and design 2"?

Key words: Solder paste, non-contact, jet printing, printing, reliability

Introduction
Surface mounting technology has come to dominate the production of commercial electronics over the last thirty years. The connection of components to metallic pads using a metallic alloy delivered onto the printed circuit board (PCB) as a suspension and a reflow step is the dominant methodology for electronics production. The demands on volume delivery and positioning accuracy for solder paste deposits are increasing as the size and complexity of circuits continue to develop in the electronics industry. Board designs that include advanced BGAs, CSPs with 0.4 mm and 0.3 mm pitch, as well as simpler 01005 and 008006 components, raise the bar for positioning demands and volume delivery and repeatability for solder paste deposits. According to the 2016 iNEMI roadmap placement accuracy for these kinds of components will reach 6 sigma placement accuracy in X and Y of 30 um by 2019[1]. This level of placement accuracy for components must be accompanied by a related accuracy for the deposit of solder paste and related fluids in order to fulfill the related increasing demands on interconnect reliability in increasingly demanding environments with respect to temperature extremes, mechanical stresses and/or production limitations[2][3]. Among the alternatives for the deposition of solder paste and other fluids on a PCB is the non-contact deposition technology jet printing, which offers advantages concerning precise volume repeatability, software control and local volume control. In this study, the question was how to perform statistically reliable robustness tests for the non-contact drop-on-demand printing of functional fluids, such as solder paste and conductive adhesives.

Jet printing
Jet printing comprises the non-contact deposition of a functional material through the transfer of momentum from a piston to the material, in this case solder paste. Jet printing on the fly is the capability of jet printing material while in motion. To jet solder paste reliably, the transfer of momentum must be made while minimizing the risks of deforming the metal alloy particles to eliminate the coining effect, thus resulting in continuous jet printing over time. A possible method of transferring momentum without contact is through very high accelerations. Coupling high accelerations with precise volumetric control enables the jet printing of a wide range of deposit sizes with a single hardware setup, see Figure 1.

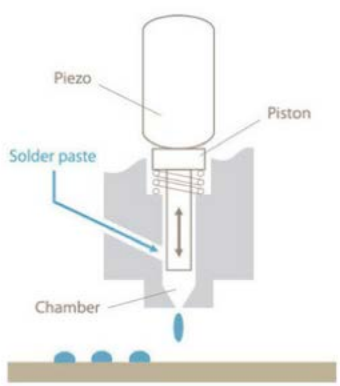

Figure 1: A schematic of a solder paste jet with an auger that feeds solder paste to the jet printing chamber and a piston that transfers momentum to the solder paste.

By controlling the material fed into a jet printing chamber, a fixed volume is created that is the basis of the jet printing process. The volume that has been transferred into the chamber is forced through the nozzle in a single shot by the volumetric displacement provided by the piezo unit. The chamber is then refilled before the next ejection. Using this principle, a variable volume of material depositions can be created without changing the frequency of the jet because the solder paste volume is precisely controlled through the feed of material into the chamber [4].There are physical limitations of the mechanics which define the functional range of this apparatus, but these are well defined and enable a wide range of outputs with a single setup at a constant speed of up to 500 Hz.

Jet printing solder paste not only allows for single deposit variation of volume, but also enables multiple pass possibilities to customize solder paste deposits. Customization can be done with respect to volume, paste height, shape, position and pad coverage which can be seen in Figure 2.

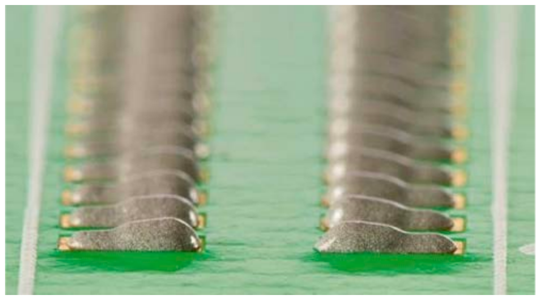

Figure 2: Examples of the control of paste height (2.5 D printing), pad coverage and volume.

The accuracy and repeatability of deposits with respect to volume, diameter, and positioning is of primary importance for any application of the technology. Therefore, an efficient and statistically sound evaluation of the reliability of jetting robustness is necessary.

The goal of this study is to develop a general method for hypothesis testing when robustness tests are performed. The main problem is to determine if there is a statistical difference between two means or proportions of jet printing devices. In this study, an example of jetting quality variation is used when comparing two jet printing ejector types that differ slightly in design. We would like to understand if the difference in ejector design can impact jetting quality by performing robustness tests. and thus answer the question, "Can jetting differences be seen between ejector design 1 and design 2"?

Compute differences of means when performing hypothesis testing

When comparing different means of material quantities obtained from the measurement of material depositions, we can use the relation

$$\frac{(\bar{x}_1-\bar{x}_2)-(\mu_1-\mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \xrightarrow{F} N(0,1). \quad (1)$$

Note that Equation 1 is used to work with continuous variables. It is assumed that each observation is independent and identically distributed, and $E(X_i) = \mu$ and $V(X_i) = \sigma^2 < \infty$. Note that $\bar{x}_i$ represents the sample mean, and μ_i represents the true population mean, if we were to jet an infinite number of deposits with different ejectors, which is obviously impossible. σ_i^2 is the true population variance, which is also unobservable in this case, and n_i is the number of observations (deposits) in each test. Since the number of observations is large, the population variance can be approximated with the sample variance, s^2. The transformation in Equation 1 gives a t-value, which can be used to evaluate if we reject our hypothesis or not. In most of the experiments, we will initially formulate that the difference between the two population means is zero, i.e. the means are equal.

Equation 1 can be understood intuitively as a sequence of random variables that converge into a standard normal distribution with mean 0 and variance 1. The null hypothesis is formulated as: *There is no difference between the means.* The alternative hypothesis is often formulated as: *There is a difference between the means.* A large difference between the observed means, when subtracting the difference between the means under the null hypothesis (which we assume to be zero in most cases), divided by the square root of their sample variances implies that there is a high probability of a difference between the means. Therefore, the null hypothesis is rejected if t-values are obtained that are either too large or too small. The t-values are selected based on the significance level α, the probability of rejecting the null hypothesis given that it is true, which is commonly referred to as critical values. Two very common significance levels are 5% and 1%, and their corresponding critical t-values are 1.96 and 2.326, respectively, if a two-sided alternative hypothesis is formulated and has more than 1000 observations. Note that the null hypothesis is rejected if $|t_{obs}| > t_{crit}$. The t-values are used because the population variance σ^2 is estimated with the observed sample variance s^2.

Hypothesis testing

Testing our hypothesis
We now would like to develop a general method for testing if there is a statistical difference between the means of two sets of measurements in the a test of process robustness. A formal approach is

1. **Null Hypothesis:**H_0: There is no difference between the jetting quality variation of ejector design 1 and ejector design 2.
2. **Alternative hypothesis:**H_A: There is a significance difference.
3. **Test statistic:** Equation 1 is used as our test statistic, but replace the population variance σ^2, with the sample variance s^2.
4. **Rejection region:** Reject H_0 if $|T| > T_{\alpha/2}$, where $T_{\alpha/2}$ is a critical value, based on the chosen significance level. T is the statistic which is obtained by using Equation 1.

Problems with our hypothesis tests
A problem that arises when using the hypothesis tests for continuous variables is if the mean of a quantity varies for iterations of identical jetting jobs, i.e. there is a significant difference between means for different jobs, although no change has been implemented in the test. In the tests described below, a generic ball grid array (BGA) board pattern is used that includes 96 individual 360 pad 0.4 mm pitch BGAs. The BGA pattern used on the test board is shown in Figure 3. The pattern was jetted on photo paper placed on a blank FR4 carrier measuring 210 by 297 mm. Diameter (area) measurements were made using a standard camera system in the jetting device, while volume measurements were made using a commercial optical solder paste measurement device. Goal diameters for the BGA deposits ranged between 210 and 270 μm.

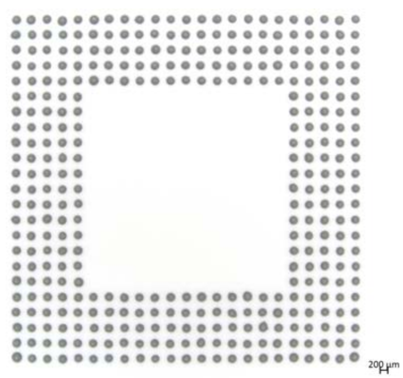

Figure 3: Generic 360 pad BGA pattern used in the jetting job.

All controllable variables have been held fixed between the tests, except the variable of interest. The reason for a significant difference in means between different jobs is unknown, but a probable explanation is that there may be many other variables that effect the resulting deposition, which the experimenter can not control. Ideally, we would like to have the deposits as homogeneous as possible, that is, observations should have the same characteristics throughout the jetting series, when the same ejector is used in the jetting job, i.e. deposits are produced with the same goal deposit size. Instead, something interesting is observed. There is no visual drift by looking at the plots, but the t-values are significant when comparing different jobs from the same ejector. The sign of the t-values appear to be random, or there is at least no observable trend when visualizing the t-values. Figures 4 and 5 illustrates this behaviour in the jetting process, based on the BGA observations. Figures 4 and 5 shows all observations in the BGA job for the diameter and volume, respectively.

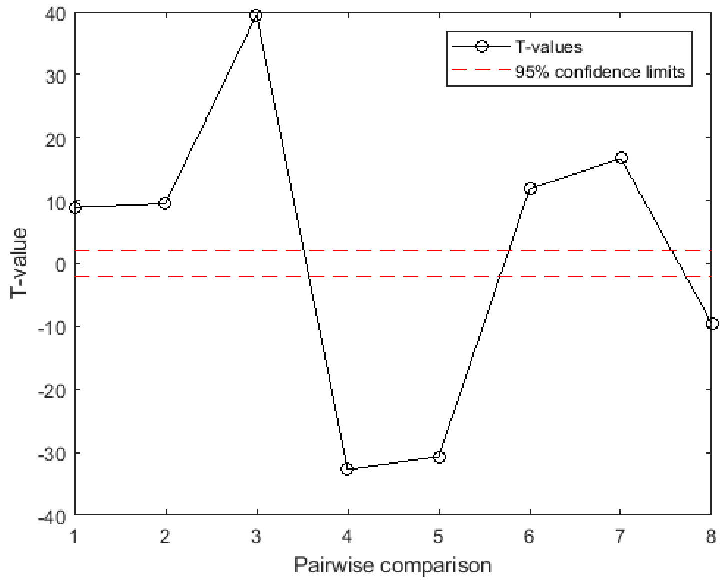

Figure 4: Pairwise t-tests of BGA jobs where we tested H_0: There is no significant difference between the different jobs versus H_A: There is a significant difference between the mean diameters of different jobs. A 5% significance level is used in the tests.

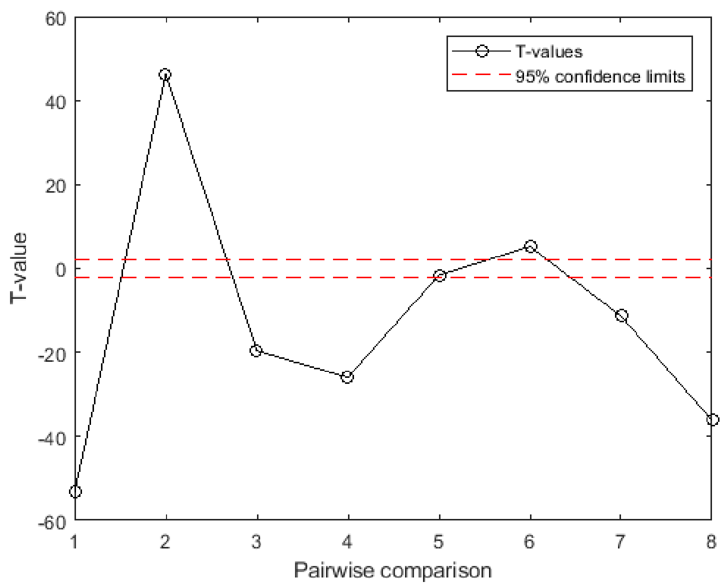

Figure 5: Pairwise t-tests of BGA jobs with the same ejector where H_0 was tested. H_0: There is no significant difference between the different jobs versus H_A: There is a significant difference between the mean volumes of different jobs. A 5% significance level is used in the tests.

When comparing different deposit sizes, it is of interest to determine if there is a significant difference between the means for each of our deposit sizes. We would also like to know the relationship of the difference in means, i.e. if one ejector design generates larger or smaller values compared to a second ejector design.

Another problem that arises in our hypothesis tests is that outliers may affect the mean, i.e. observations that are distant from other observations. In general, one should be careful to remove outliers if the cause of the outliers is unknown. Since we assume that the jetting process within a job is a stochastic process, with mean around the true mean and constant variance, one could in consultation with the experimenter remove outliers if we know that the outliers are random occurrences, equally likely to occur in each of the tests. In Figure 4 and 5, it can be seen that some observations are distant from the the rest of the observations in the BGA job.

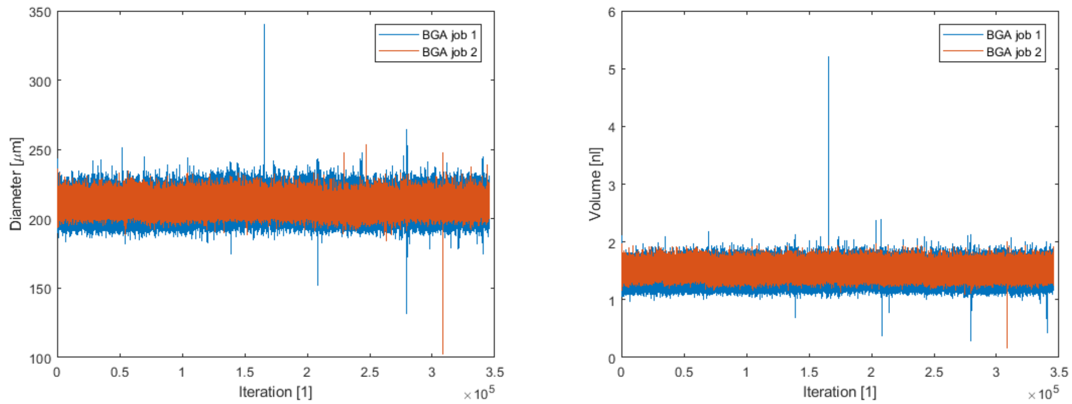

Figure 6: Plot of the a) diameter and b) volume for two identical BGA jobs

This relation can be used to develop a formal method to determine if there is a significant difference between means in a robustness test.

Formal method for testing

We now wish to develop a method for hypothesis testing to determine if the means of a robustness test differ significantly. Since it is assumed that the jetting process is a stochastic process, we want to minimize the risk of rejecting the null hypothesis due to randomness in data, as well as other confounding variables that may affect the result.

We are interested in investigating if there is a significant difference between means in a robustness test for each deposit diameter. In this case, we want to compare if the diameters and volumes for deposits with goal diameters of 210μm, 225μm, 240μm, 250μm and 270μm differ between the two ejector designs.

The method defined in Section 3 may be used again to perform our hypothesis tests between the ejector designs when comparing deposit sizes. In order to avoid randomness in the data, a restriction is added to the test. In order to minimize the risk of having randomness in our data affecting the t-values, we would like that all the t-values must be significant for the different deposit sizes, as well as having the same sign for the t-values.

Figure 7 shows that all the means for the second test are higher than those for the first test. When performing t-tests for the different ejector designs, we find that all means for each deposit size comparison differ significantly from each other.

It is also important that the order of jetting does not affect the outcome. Therefore, the same experiment is repeated using another ejector. In the new experiment, the order of the experiments is switched, i.e. reverse the shooting order for each test.

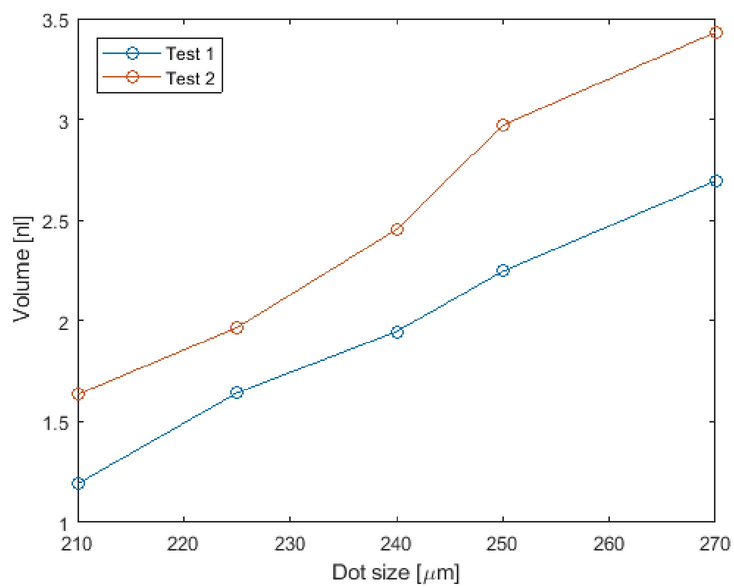

Figure 7: Plot of the volume means for the different ejector designs, when comparing the pairwise deposit sizes.

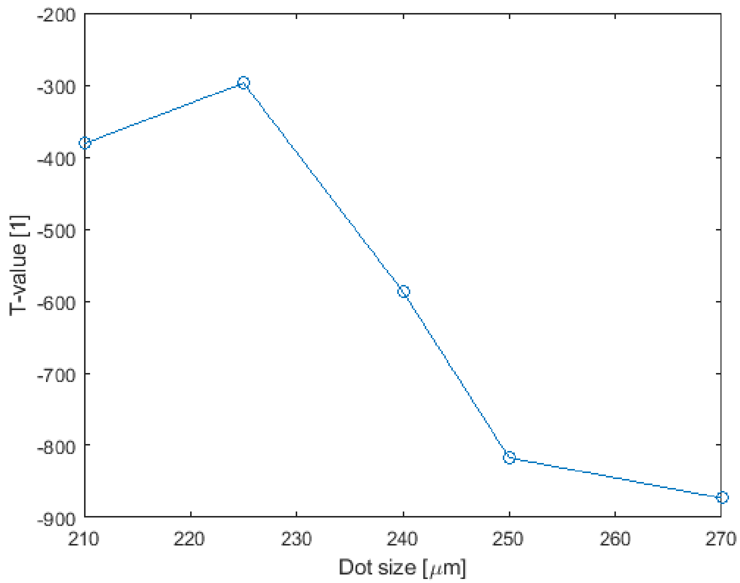

Figure 8: Pairwise t-values for the mean volumes. It is found that the null hypothesis can be rejected, i.e. reject that there is no difference between the means at 1% significance. This is because the magnitude of all the t-values are larger than our critical value, and all the t-values have the same sign.

Hypothesis testing with the p-value approach

In order to compare if there is a significant difference between missing volumes, i.e. deposits that are smaller than a certain threshold, the Central Limit Theorem (CLT) can not be used since the probability of missing is very low (close to zero). There is also a much easier way to perform hypothesis tests when we work with sums of binary variables. Note that a missing volume is a Bernoulli trial, with probability of success (missing) being close to zero. The probability of success for a Bernoulli variable is

$$P(X = x) = p^x(1-p)^{1-x} \quad \text{for} x \in \{0,1\}. \tag{2}$$

The sum of the Bernoulli trials follow a Binomial distribution with parameters n, p, and the sum of the number of missing volumes $Y = \Sigma X_i$ is distributed as

$$P(Y = y) = \binom{n}{y} p^y(1-p)^{n-y}, \quad y = 0,1,2,\dots,n. \tag{3}$$

The expected value of this distribution is np and variance $np(1-p)$. Note that $p = \Sigma x_i/n$, i.e. a simple average of the binary outcomes.

The $p-$value approach involves determining the probability of obtaining a more extreme statistic, given that the null hypothesis is true. If one has data from previous tests of the missing volumes given a certain solder paste, then the probability of missing can be estimated with the obtained statistic from the previous tests. Otherwise, one can assume that the null hypothesis in this case is the missing estimate obtained in the first test. Here, we could say that we assume that the estimate obtained from the first test is the true parameter value. We compare the probability of obtaining our second estimate, given that the first estimate is true, by calculating the probability of observing a more extreme statistic in the direction of the alternative hypothesis. If the $p-$value is small (less than α), it is unlikely that the tests have the same parameter values, and the null hypothesis is rejected.

Let us first understand that Binomial distribution can be used to compare missing volumes. Recall from Equation 2 that a random deposit is a Bernoulli experiment with two outcomes, success or failure (success can also be something bad, in this case missing). In order to understand the concept of the Bernoulli variables, let us consider the example of a coin. The probability of heads is 0.5, and tails is 0.5, since it is equally likely for each outcome. If we use the Bernoulli formula and are interested in the probability of success, we get

$$P(X = 1) = 0.5^1(1 - 0.5)^{1-1} = 0.5.$$

The probability of no success (tails) is

$$P(X = 0) = 0.5^0(1 - 0.5)^{1-0} = 0.5.$$

If n independent Bernoulli trials are performed, the sum of the number of successes turns out to follow the Binomial distribution. If the coin is flipped $n = 3$ times, the probability of $Y = \Sigma X_i$ for the different outcomes are now

$$P(Y = 0) = (1 - 0.5)^3 = 0.5^3$$

$$P(Y = 1) = 0.5 \cdot (1 - 0.5)^2 + (1 - 0.5) \cdot 0.5 \cdot (1 - 0.5) + (1 - 0.5)^2 \cdot 0.5 = 3 \cdot 0.5 \cdot (1 - 0.5)^2 = 3 \cdot 0.5^3$$

$$P(Y = 2) = 0.5^2 \cdot (1 - 0.5) + 0.5 \cdot (1 - 0.5) \cdot 0.5 + (1 - 0.5) \cdot 0.5^2 = 3 \cdot 0.5^3$$

$$P(Y = 3) = 0.5^3.$$

Using the Binomial theorem, the reader can verify that the formula

$$P(Y = y) = \binom{n}{y} p^y (1 - p)^{n-y} = \frac{n!}{(n-y)!y!} p^y (1 - p)^{n-y}$$

gives the same result. Another way of thinking on the sum of the number of successes is to consider it as the joint probability by multiplying the Bernoulli trials together

$$\prod_{i=1}^{n} p^{x_i}(1 - p)^{1-x_i} = p^{\Sigma x_i}(1 - p)^{n-\Sigma x_i} = p^y(1 - p)^{n-y}.$$

Since the Bernoulli trials are independent, the trials may be multiplied in order to get the joint probability distribution, but the binomial coefficient, $\binom{n}{y}$, must also be added in order to calculate the joint probability of the number of successes, and not only a specific order of the outcome. Recall the example of flipping the coin $n = 3$ times, where not only the outcome must be considered, but also the number of ways that a specific outcome can occur.

Begin by considering the example of calculating the p-value, where we assume that we have performed a robustness test. In the first parameter settings, 300,000 deposits were jetted, and the number of missing volumes were $\Sigma x_i = y = 10$. After implementing a change in the ejector, another series of 300,000 deposits were jetted with 8 missing. It is assumed that the estimate obtained in the first series is the true parameter value. Hence, $\hat{p}_1 = 10/300000$ and $E[Y] = 300000 \cdot \hat{p} = 10$. We would now like to calculate the probability that

$$|Y - 10| \geq |8 - 10| = 2 \text{ given that } Y \sim Bin(n, \hat{p}_1).$$

We can calculate this event by

$$P(Y \leq 10 - 2 \cup Y \geq 10 + 2)$$

$$= P(Y \leq 8 \cup Y \geq 12)$$

$$= P(Y \leq 8) + P(Y \geq 12)$$

$$= 1 - P(8 < Y < 12).$$

Using a computer, the p $-$value is in this case 0.6360, rounded to four digits. From Figure 9, it is observed that approximately 63% of the observations are outside the interval $8 < Y < 12$, and hence, we can not make any conclusions about the sample since the P $-$value is above our significance level α (0.05 or 0.01).

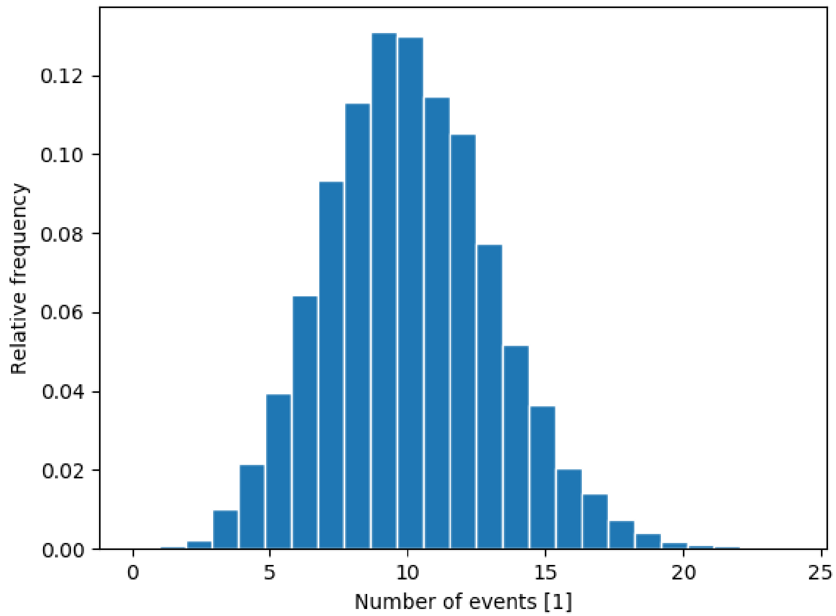

Figure 9: Simulation of the Binomial distribution with parameters $N = 300000$, $p = 10/300000$.

How many observations do we need?
In the field of statistics, one would like to have as many observations as possible in order to draw conclusions about the data. In statistics, as well as many other fields of science, one can never say that *"we know that something is true"* or *"accept the alternative hypothesis"*. Theories may only be falsified by rejecting the null hypothesis, but conclusions can not be reached about the alternative hypothesis.

One rule of thumb when using the central limit theorem

$$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \xrightarrow{F} N(0,1),$$

is to have at least 30-50 observations. The uncertainty decreases when the sample size is increased. We seek to obtain as much information as possible at minimum cost. This is commonly referred to as *experimental design*. Let us now develop a common method for selection of sample sizes in our experiments. Let us go back to the question, *How many measurements should be included in the sample?*. The experimenter can indicate the desired accuracy by specifying a bound on the error of estimation. Suppose that the desired deviation from the true mean is U, with probability 95%. Recall that the critical t-value was 1.96 ≈ 2. Since approximately 95% of the sample means will lie within 2σ of μ in repeated sampling, we have

$$\frac{2\sigma}{\sqrt{n}} = U \Leftrightarrow n = \frac{4\sigma^2}{U^2}.$$

Let us approximate the minimum sample size given a certain accuracy. The mean of the diameter in previous tests for 210 μm is 180 μm, and mean for the volume for the same deposit-size is 1.2 nl. The sample variances obtained in a previous test for the diameter and volume is 45.45 μm and 0.0162 nl, respectively. Tables 1 and 2 show how the sample size changes as the desired precision in our estimates is increased.

Table 1: Selection of sample size based on diameter data (210 μm)

U (deviation from the true mean) [μm]	5	2.5	1	0.5	0.025
Sample size [1]	7	28	175	699	279 362

Table 2: Selection of sample size based on volume data (210 μm)

U (deviation from the true mean) [nl]	0.3	0.2	0.1	0.05	0.01
Samplesize [1]	1	2	7	25	623

Summary

It is necessary to be careful when calculating the minimum number of observations required given a certain accuracy. As mentioned earlier, the central limit theorem deals with independent and identically distributed random variables, although the latter condition can sometimes be relaxed. One problem that arises throughout the statistical tests is that there are many variables that the experimenter can not control, such as the state of the system, i.e. if the system is unstable during a certain period of jetting, and stable during another period of jetting. Different states of jetting could be modelled and included in a subsequent effort. In previous tests, where jet printing has been tested, 300 000 deposits have been enough to reject the null hypothesis. Although the number of observations could easily be reduced from a theoretical perspective as illustrated in the tables, one has to take these other factors, such as uncontrollable variables, dependence of observations etc., into account when designing the experiment and choosing the sample size.

Another important subject when designing an experiment, although it might seem obvious to many, is to hold as many variables fixed between the experiments in order to avoid confounding variables to affect the outcome of the experiment. For instance, when testing jet printing with ejectors with specific design differences, one would like to have a majority of test factors constant for both designs, for example solder paste, actuation profile et cetera. Holding all controllable variables fixed except the variables of interest between tests may increase the internal validity of the experiment. Also, since the uncontrollable variables in the jetting process are assumed to be random, we assume that our result will converge to some value as the number of observations is increased, while using different ejectors.

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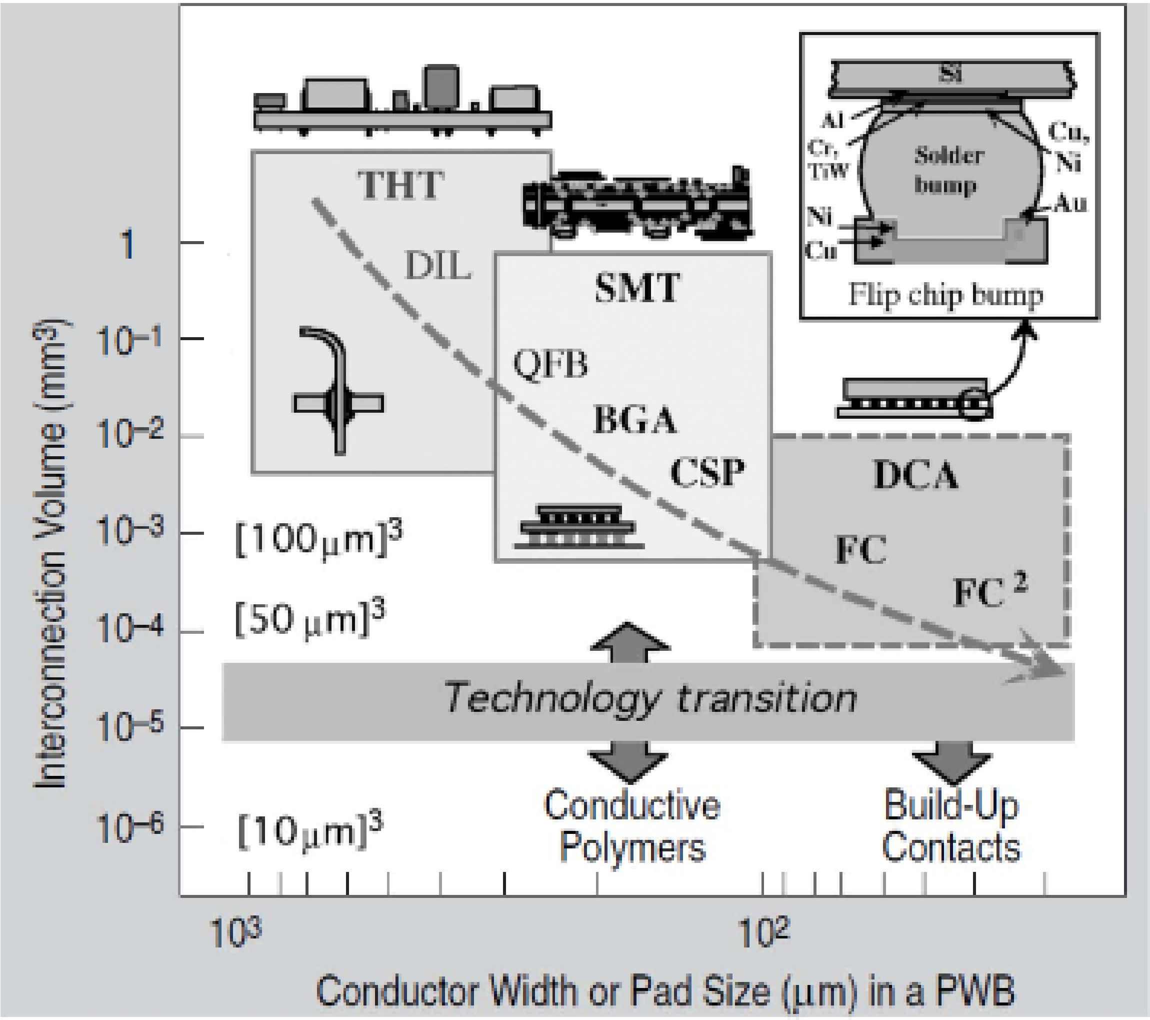
Background

Deposition of material has to fulfill rigorous demands of customers concerning

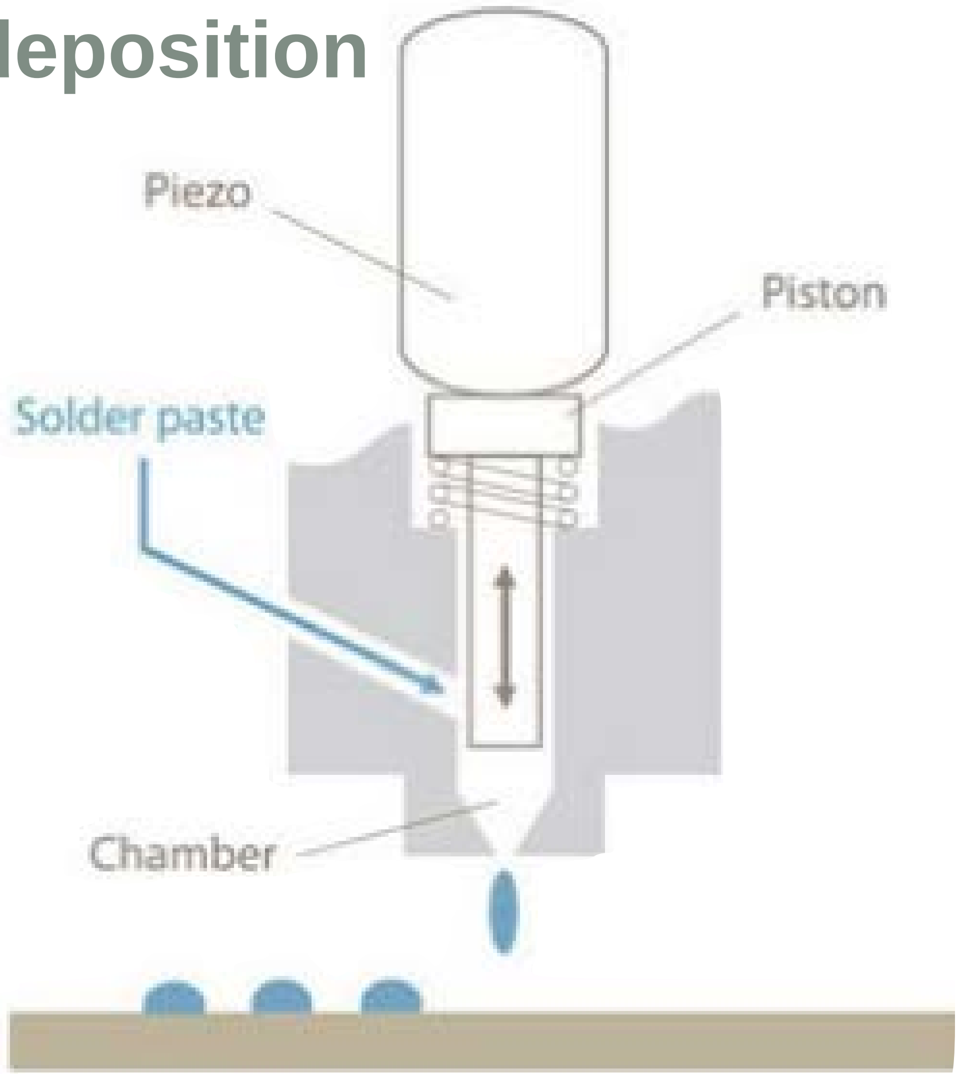
- *volume reliability*
- *positioning accuracy*
- *presence (sic!)*

How many shots are required to ensure robust results when changes can occur at the ppm level?

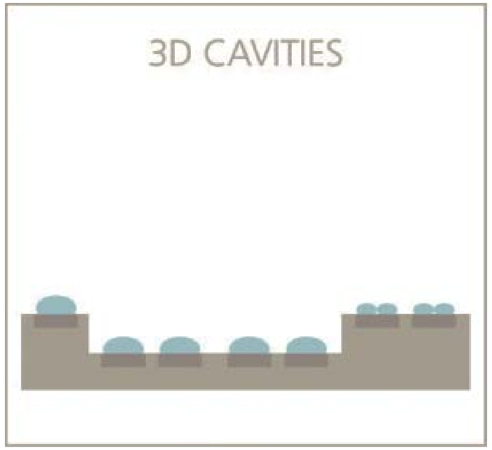
Industry demands

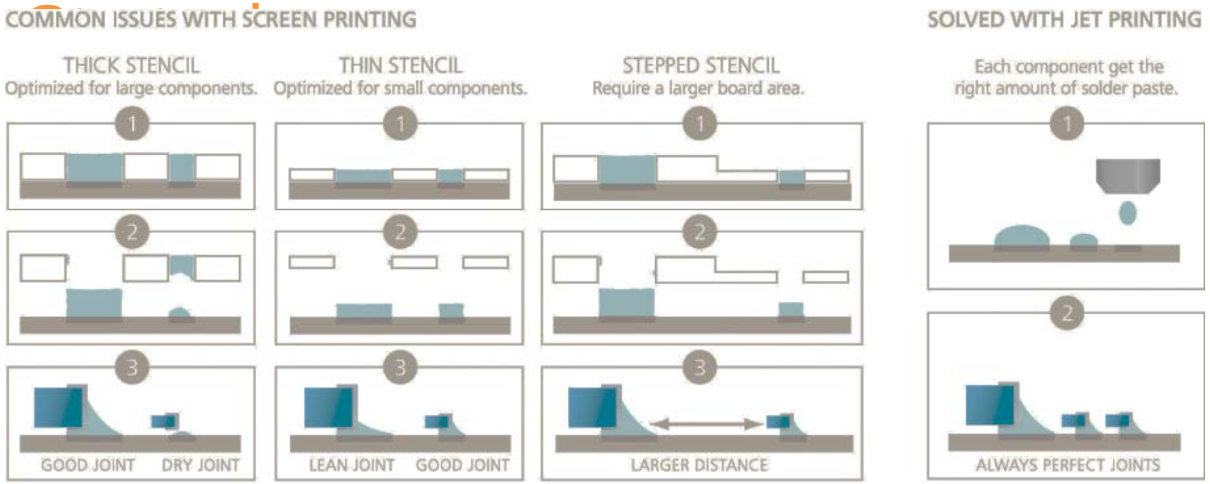


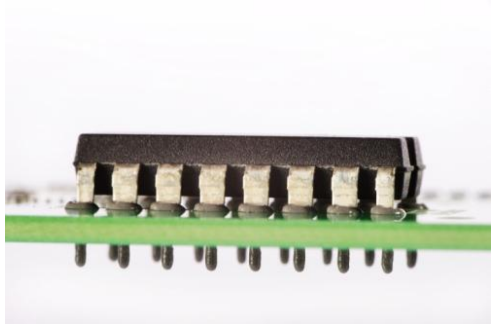
Non-contact deposition

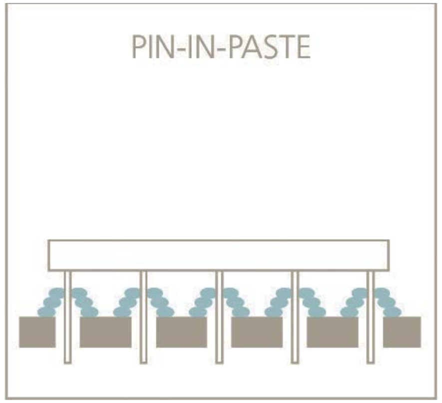


Non-contact deposition

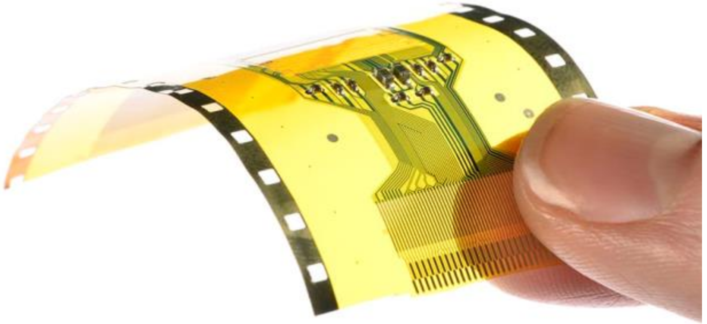




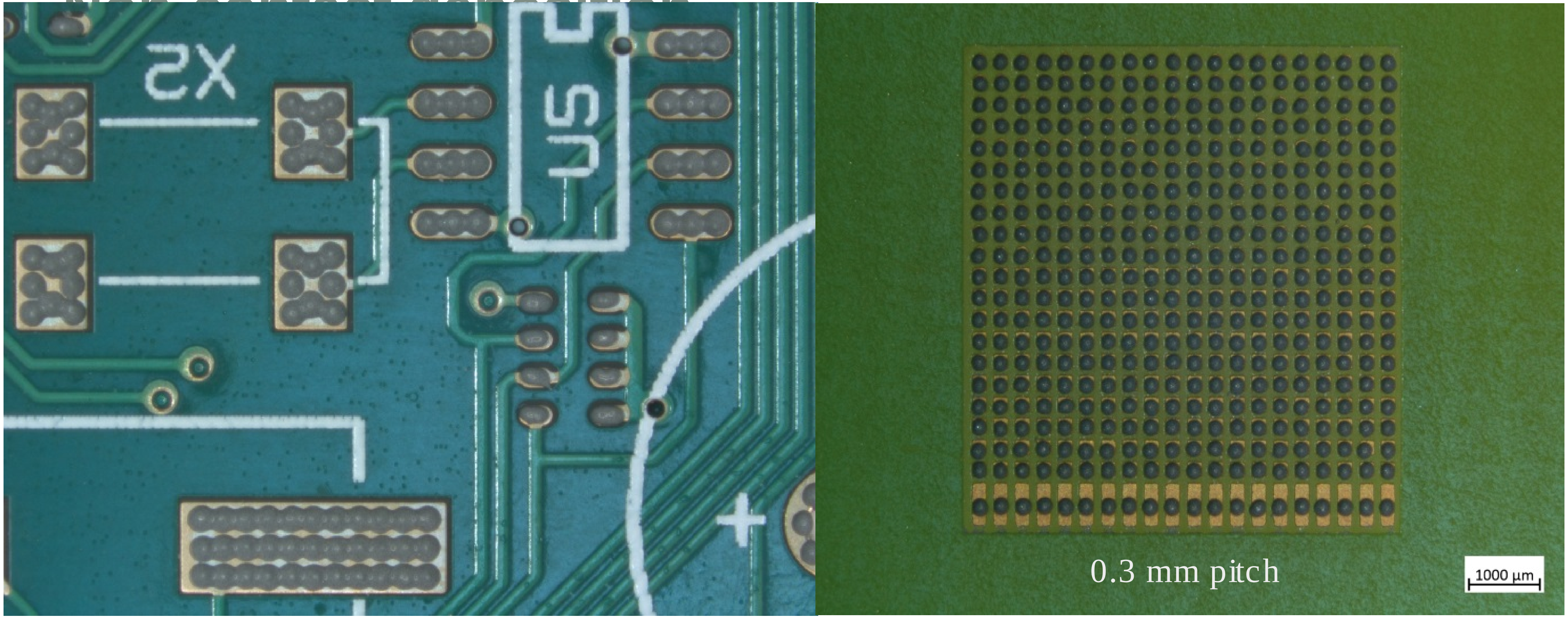


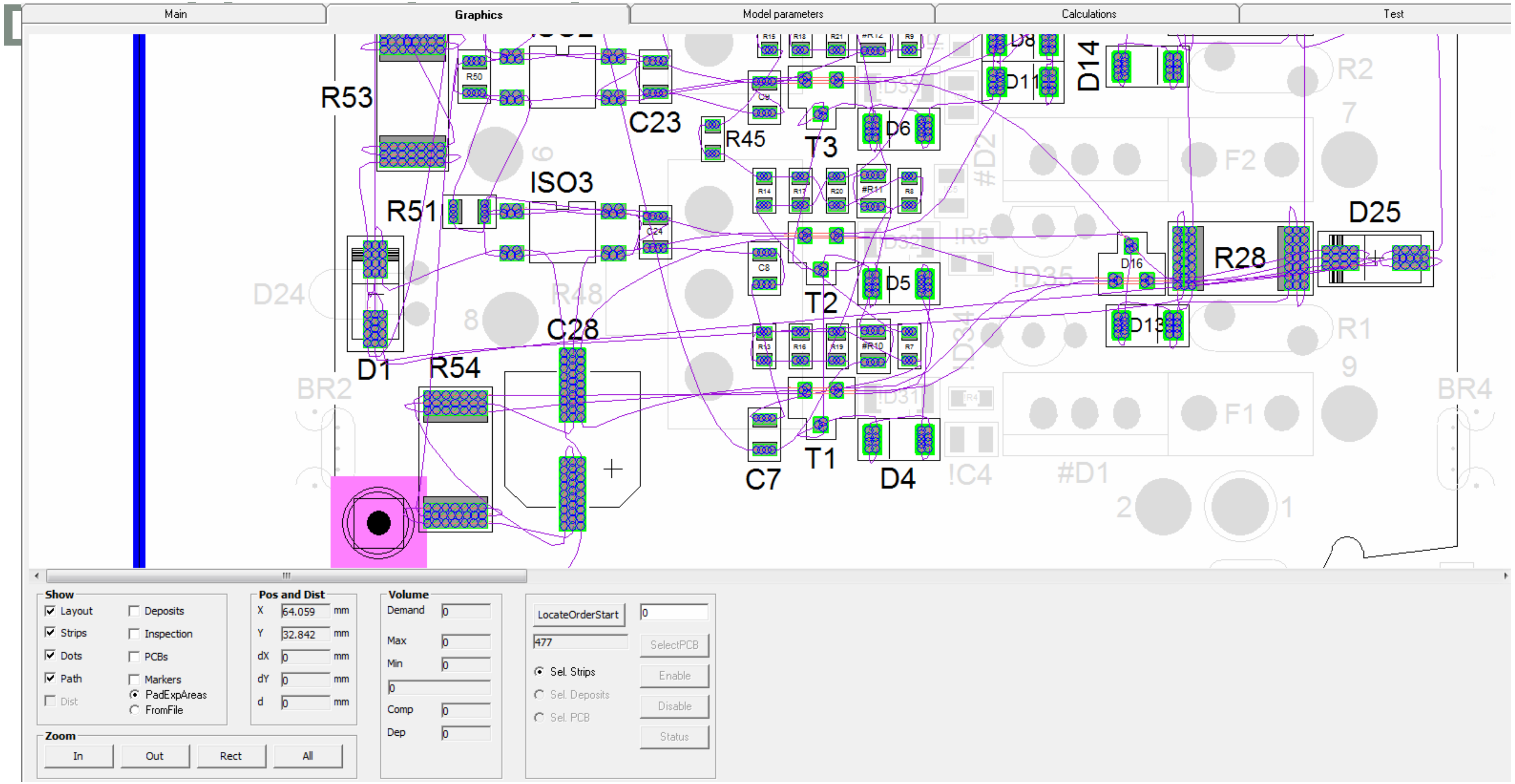


Add-on strategies



Non-contact deposition

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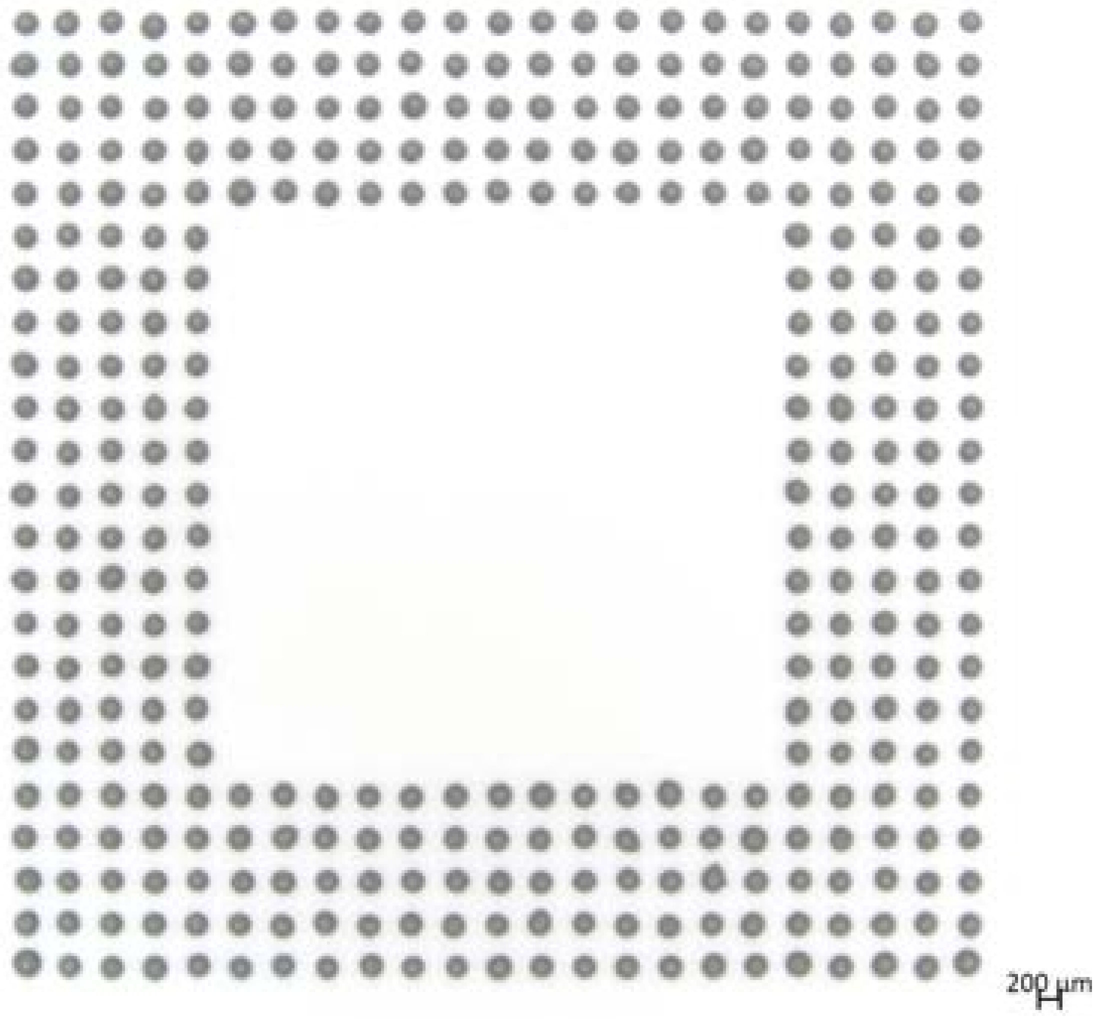
Disable
Status
Zoom
In
Out
Rect
All

Test pattern



Statistics

Central Limit Theorem

$$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \xrightarrow{F} N(0,1)$$

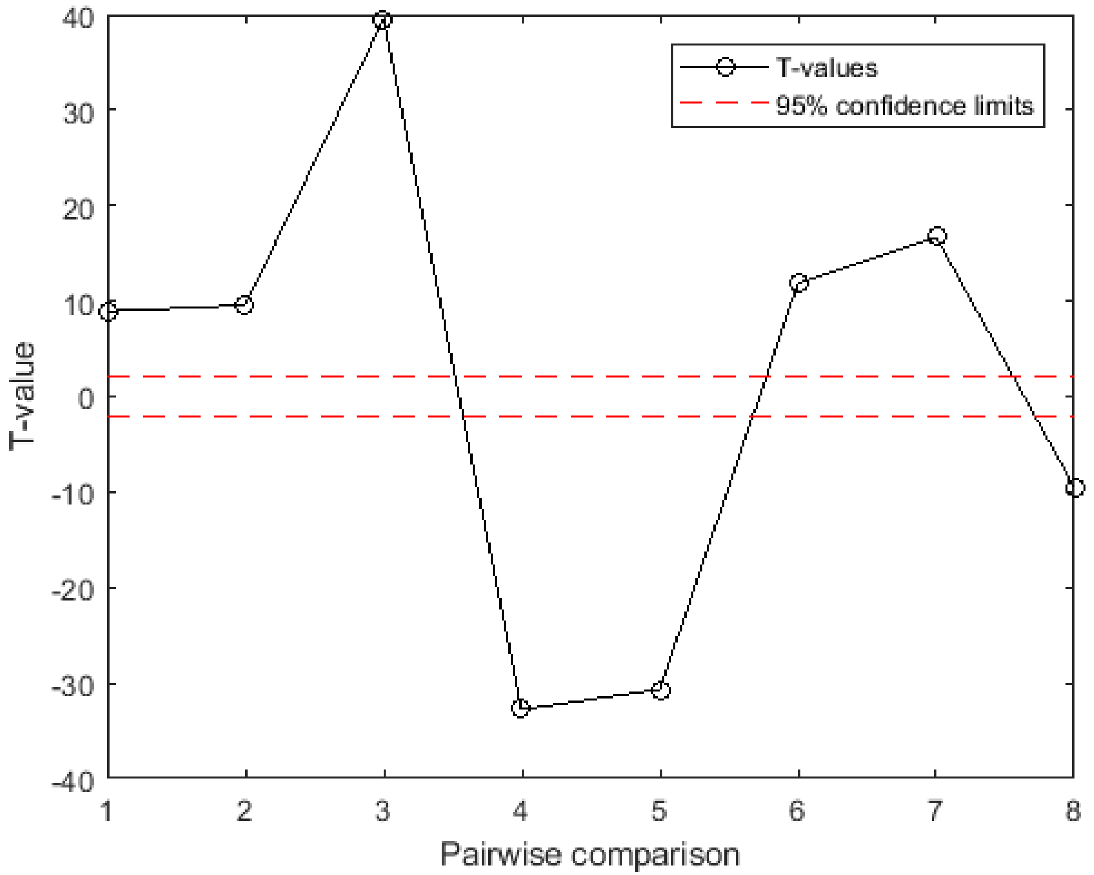
Statistics

Hypothesis testing

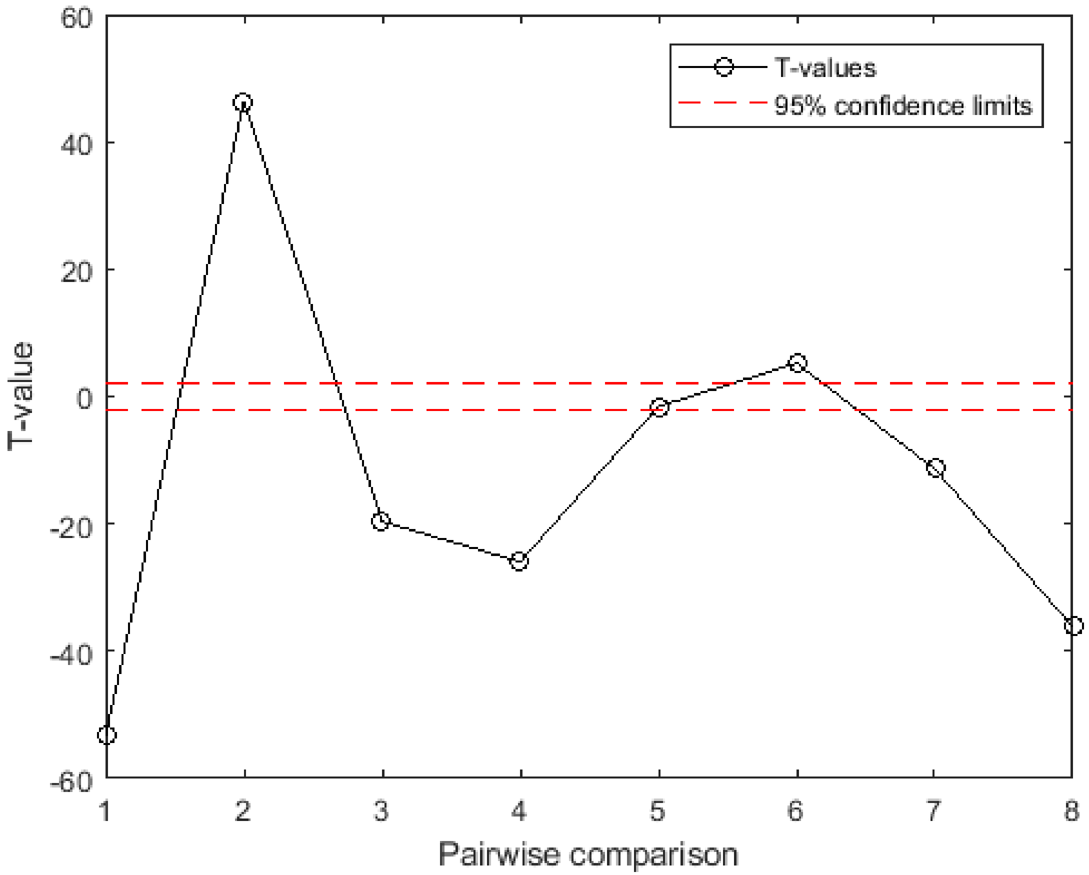
1. **Null Hypothesis:** H_0 : There is no difference between the jetting quality variation of ejector design 1 and ejector design 2.
2. **Alternative hypothesis:** H_A : There is a significance difference.
3. **Test statistic:** Equation 1 is used as our test statistic, but replace the population variance σ^2, with the sample variance s^2.
4. **Rejection region:** Reject H_0 if $|T| > T_{\alpha/2}$, where $T_{\alpha/2}$ is a critical value, based on the chosen significance level. T is the statistic which is obtained by using Equation 1.

Hypothesis testing

Statistics

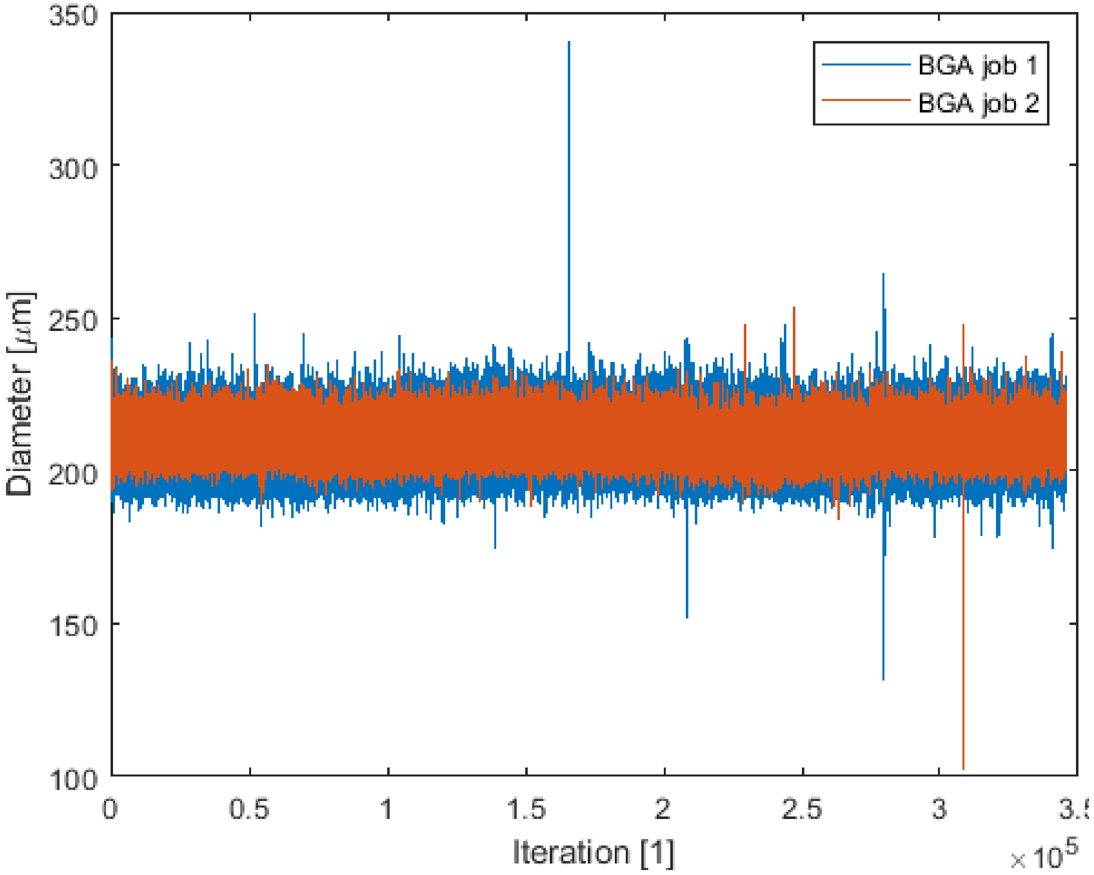


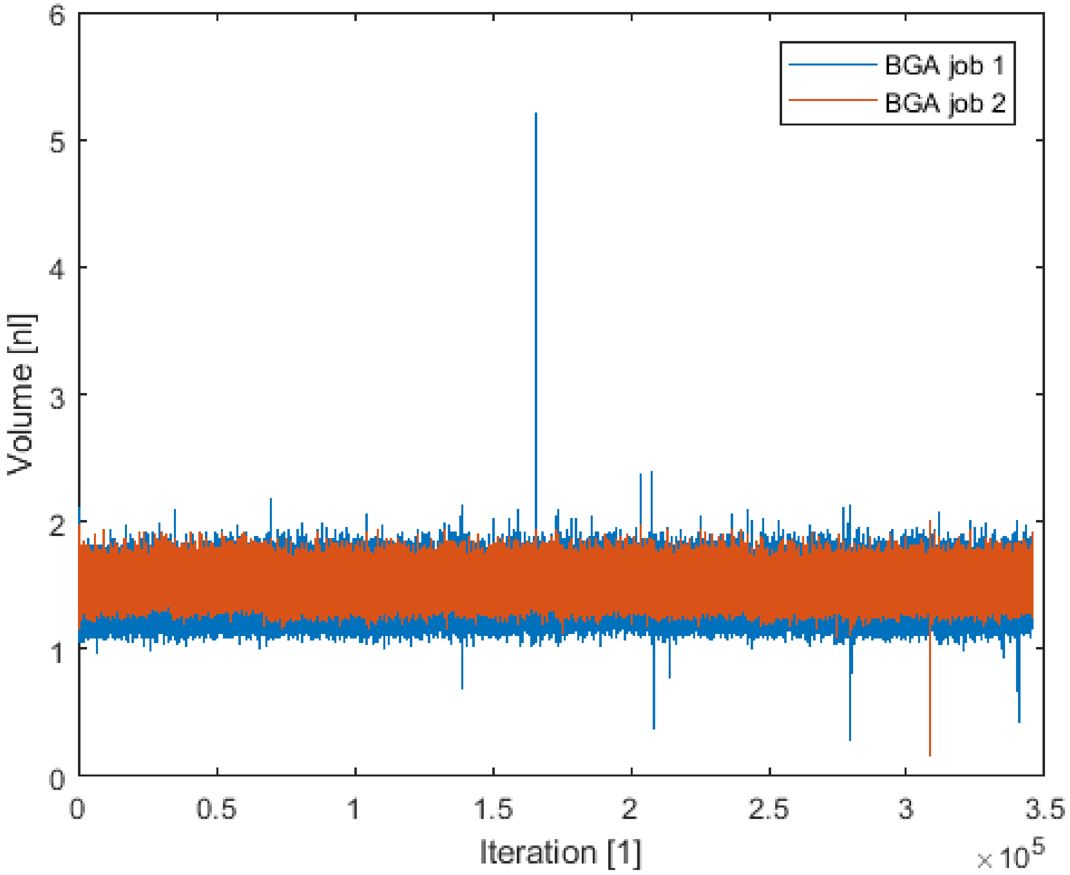
H0: There is no significant difference between the different jobs versus HA: There is a significant difference between the mean diameters of different jobs.



H0: There is no significant difference between the different jobs versus HA: There is a significant difference between the mean volumes of different jobs.

Statistics


BGA job 1
BGA job 2
350
300
250
200
150
100
Diameter [μm]
0
0.5
1
1.5
2
2.5
3
3.5
Iteration [1]
× 10^5


BGA job 1
BGA job 2
6
5
4
3
2
1
0
Volume [nl]
0
0.5
1
1.5
2
2.5
3
3.5
Iteration [1]
× 10^5

Statistics

Bernoulli test

$$P(X = x) = p^x (1-p)^{1-x} \quad \text{for } x \in \{0, 1\} \ .$$

$$P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}, \quad y = 0, 1, 2, \ldots, n.$$

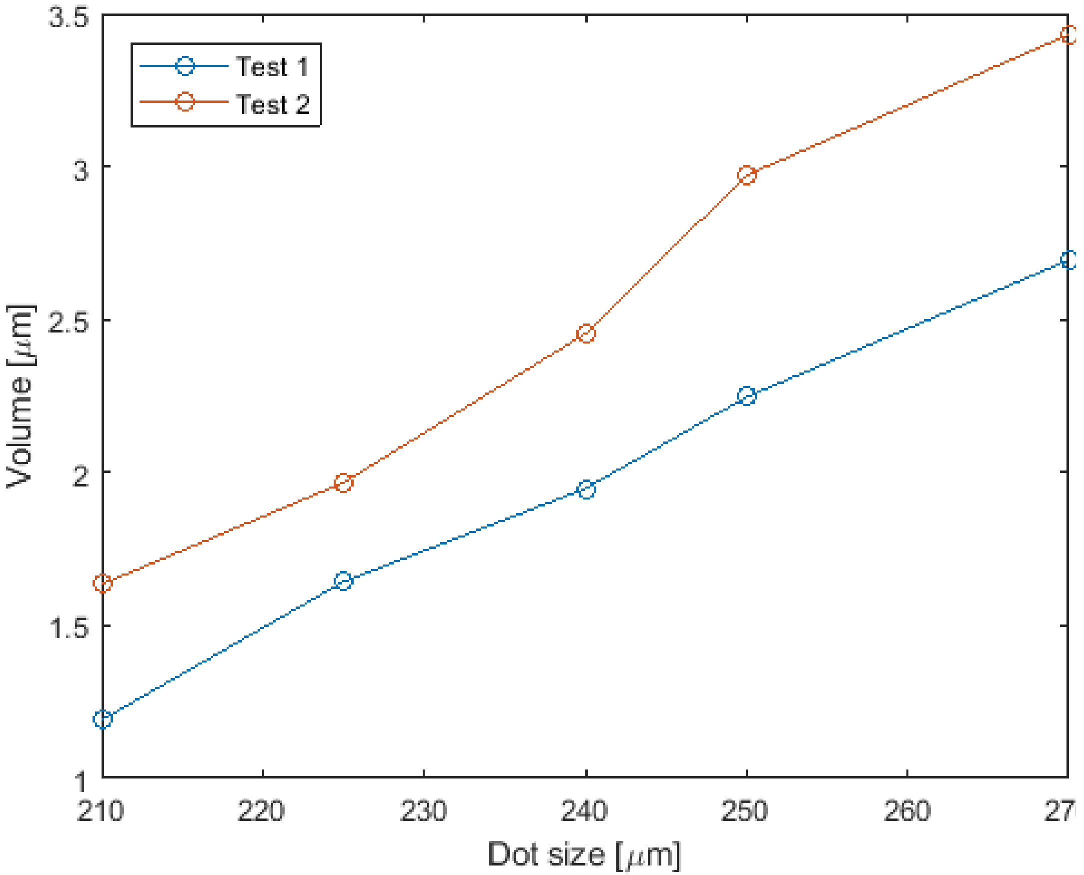
Statistics

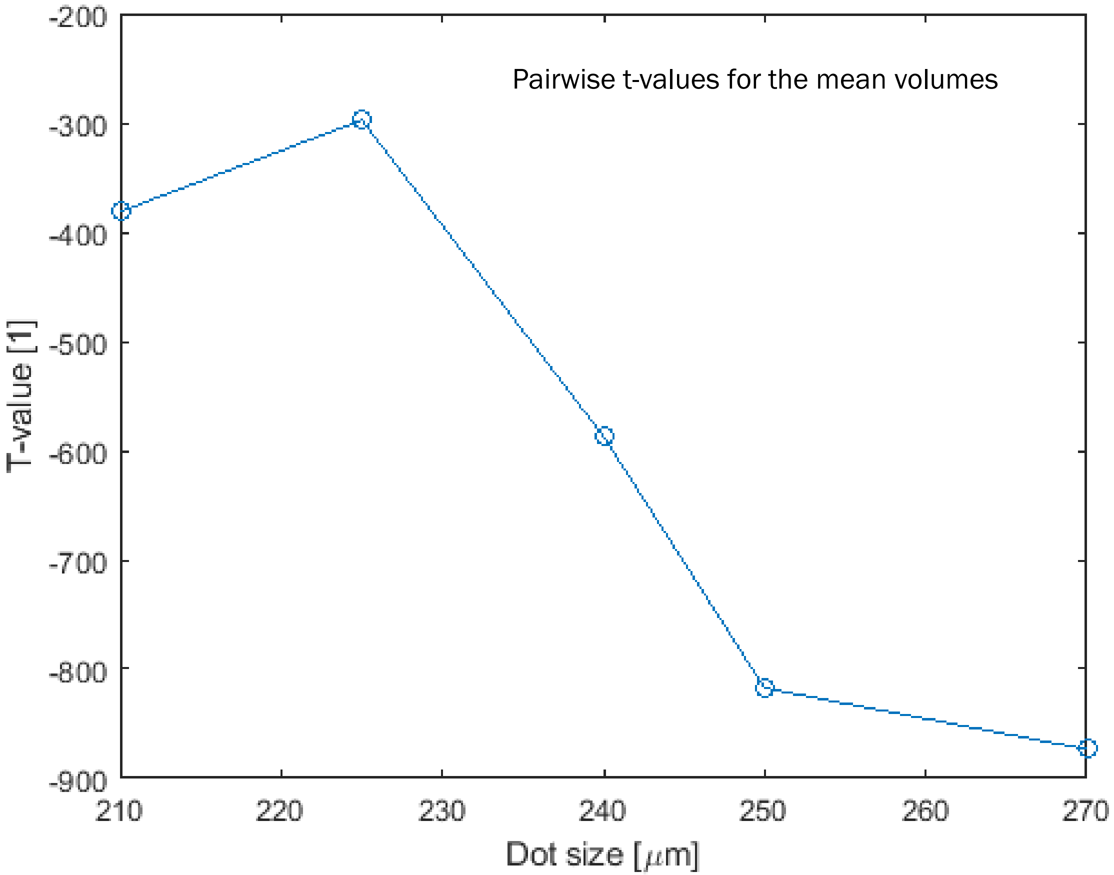
Central Limit Theorem

$$\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \xrightarrow{F} N(0,1)$$

$$\frac{2\sigma}{\sqrt{n}} = U \Longleftrightarrow n = \frac{4\sigma^2}{U^2}$$

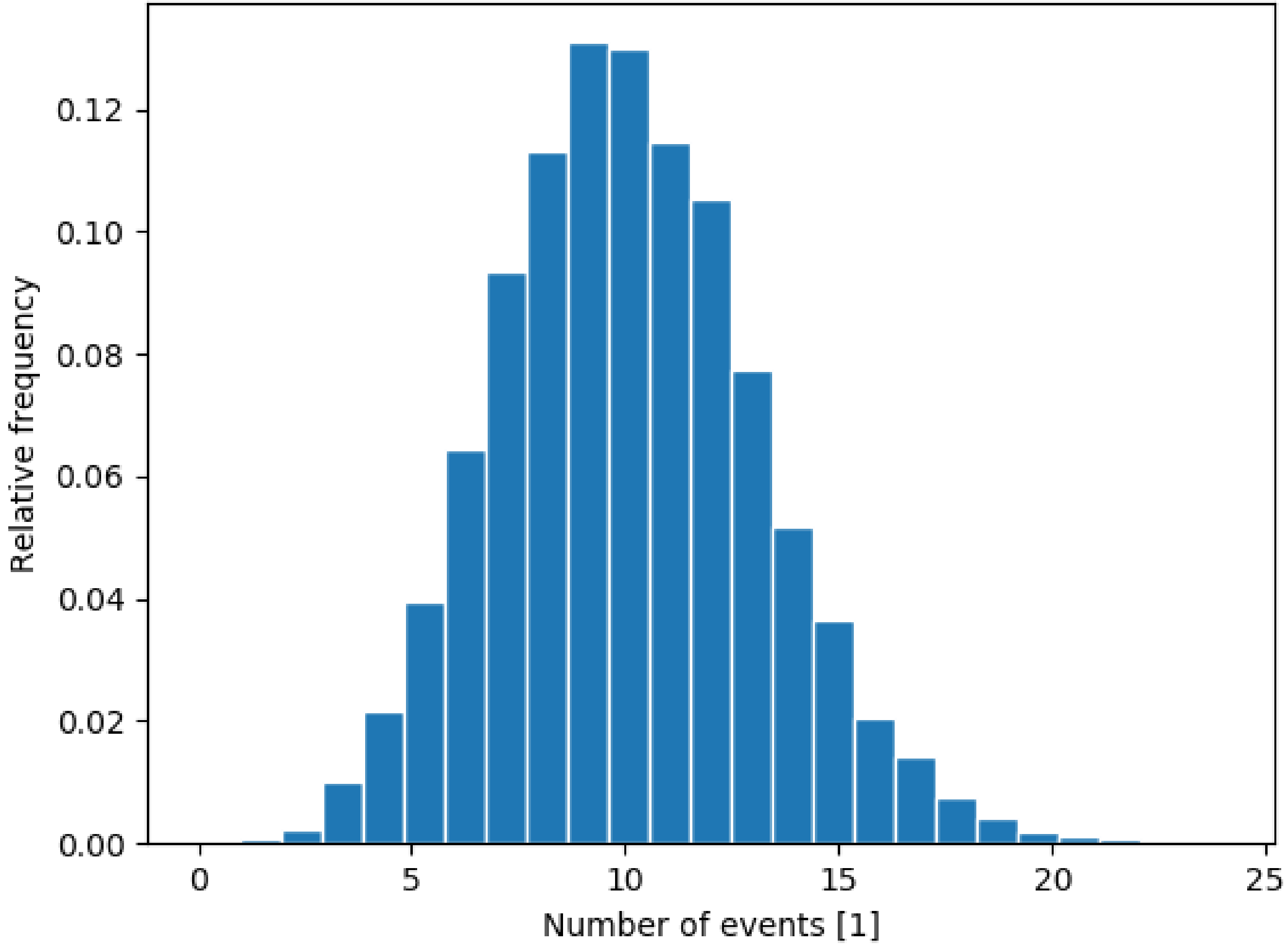
Statistics


3.5
3
2.5
2
1.5
1
Volume [μm]
Test 1
Test 2
210
220
230
240
250
260
27
Dot size [μm]


Pairwise t-values for the mean volumes
-200
-300
-400
-500
-600
-700
-800
-900
T-value [1]
210
220
230
240
250
260
270
Dot size [μm]

Statistics

Simulation of Binomial distribution



Statistics

Sample size

U (deviation from the true mean) $[nl]$	0.3	0.2	0.1	0.05	0.01
Sample size [1]	1	2	7	25	623

U (deviation from the true mean) $[\mu m]$	5	2.5	1	0.5	0.025
Sample size [1]	7	28	175	699	279 362

Summary

- Central limit theorem is used to estimate necessary sample size for non-contact deposition

U (deviation from the true mean) $[nl]$	0.3	0.2	0.1	0.05	0.01
Sample size [1]	1	2	7	25	623

U (deviation from the true mean) $[\mu m]$	5	2.5	1	0.5	0.025
Sample size [1]	7	28	175	699	279 362

THANK YOU!!

Any Questions?