# A Time Dependent Analytical Analysis of Heat Transfer in A PCB during A Thermal Excursion

By

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A great deal of work has already been done to determine the equilibrium temperature of a PCB when exposed to a heat source such as the thermal environment of reflow soldering. This study will go beyond an equilibrium condition and explore the temperature-time distribution of the board when a variable temperature heat source is applied to both outer surfaces. For simplicity, the model will be a two-sided board. Obviously, the model board has two material interfaces. An interesting observation is that anywhere within the board, including the material interface, thermal energy must be conserved. There is not a similar requirement for the temperature. Consequently, at the material interfaces we can expect the thermal properties' of the board to change in a profound manner. A similar situation occurs when a fluid passes through a shock wave. This will be reflected in such board properties as the thermal stresses in the various layers and the resulting warp. This phenomenon also explains and quantifies why a thermal shock can be devastating while a slow temperature rise to the same endpoint may well be tolerated.

The analysis will use a one dimensional, time dependent model i.e. there are two independent variables. This necessitates a partial differential equation to describe the temperature variation within the board. The boundary conditions are the outer temperature of the board, which is the temperature of the heat source on both outer surfaces. The third boundary condition is at the copper epoxy interface where conservation of thermal energy is required.

## Introduction

Techniques for measuring the surface temperature of a PCB have been available for some time. Measurements for assessing the temperature at various positions in the laminate are at least problematic. Inserting thermal couple wires well into a thin laminate will more than often distort the temperature as they now become a part of the thermal mass of the system; the so called Helmholtz effect ("to some extent the tool used to measure any physical quantity will disturb the measurement"). In this case, the thermal couple wire will locally conduct heat away from the laminate cooling that area of the package. The larger the thermal couple the greater the issue. In situations of this nature, it often is best to use analytical models to estimate the value of the desired measurement, i.e. the internal temperature history of a PCB as it goes through a heat excursion. If the temperature history distribution in the PCB package can then be estimated, the internal shear stress can be quantified. Once this is accomplished, the likelihood of forming an internal delamination can be established. Developing such a procedure and using it to determine the internal laminate history as the PCB is exposed to a temperature ramp such as in reflow, is the objective of this analysis.

## Analysis

To simplify the analysis, the investigative package will be a symmetrical double-sided PCB. The copper thickness is defined to be 1.7 mils. The thickness of the internal laminate is 60 mils and composed of FR-4, see Figure 1. The PCB will be exposed to several temperature ramps all ending at $600^{0}$F.

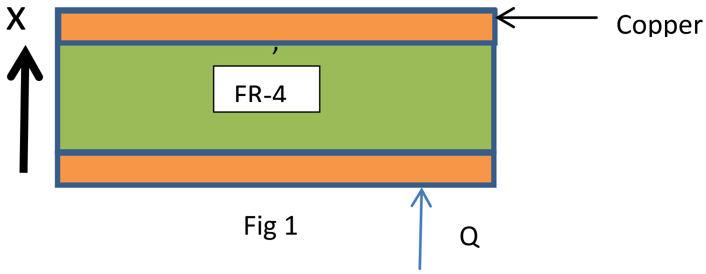

Fig 1

It will be assumed that the board has been preheated to a uniformed temperature of $300^0$F before the temperature ramp

According to first principals, heat conduction through a solid is governed by Fourier's Law of heat conduction. The assumption is made that the heat flow is one-dimensional consequently, there are two independent variables: time and position. For one dimensional, time dependent heat transfer Fourier Law becomes

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \qquad (1)$$

A unique solution requires three boundary conditions:

At t=0, T=$300^0$F for the entire board

At x=0, T is specified by the ramp temperature

Along the axis of symmetry of the board, $\frac{\partial T}{\partial x} = 0$

Unfortunately, since the second geometric derivative of temperature appears in (1), the first derivate must be continuous. Conservation of energy requires a consistent heat flow rate across the interface of copper and FR-4. Since the thermal properties of the material change at the interface, conservation of energy requires a discontinuous adjustment in the temperature gradient. To avoid the issue we will introduce the transformation (see Reference 1)

$X=k\,\beta$ (2)

Making β, the new geometric variable.

After making the transformation equation (1) becomes

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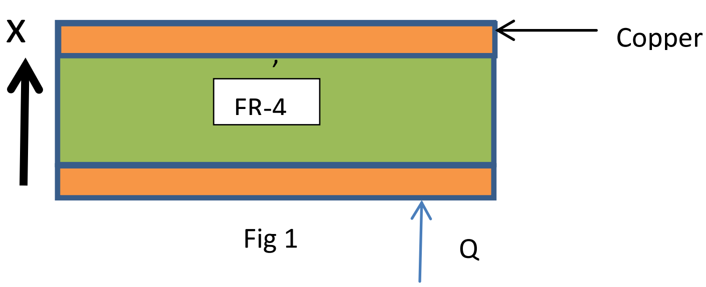

Fig 1

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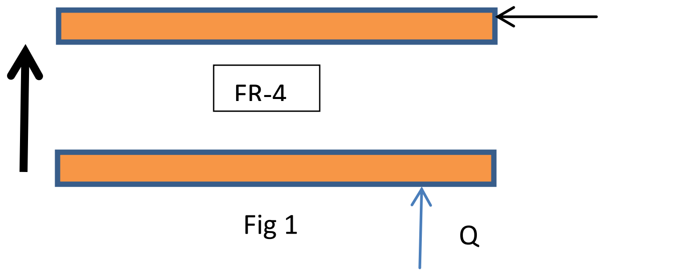


Fig 1

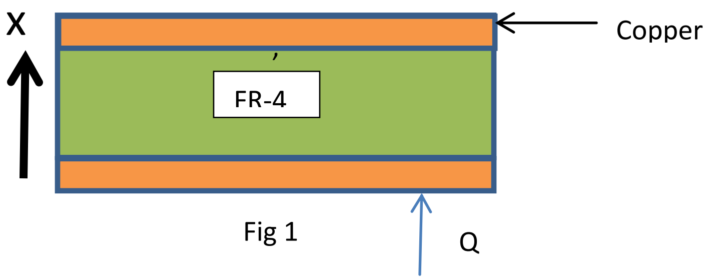


Fig 1

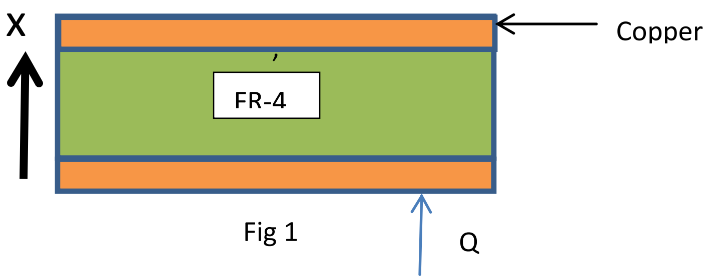


Fig 1

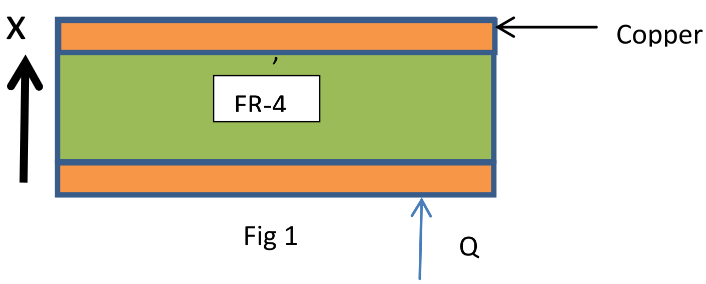


Fig 1

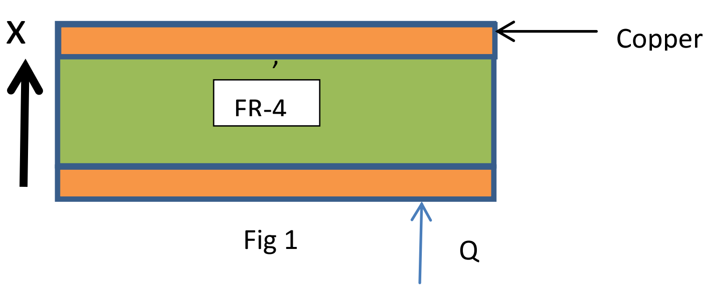


Fig 1

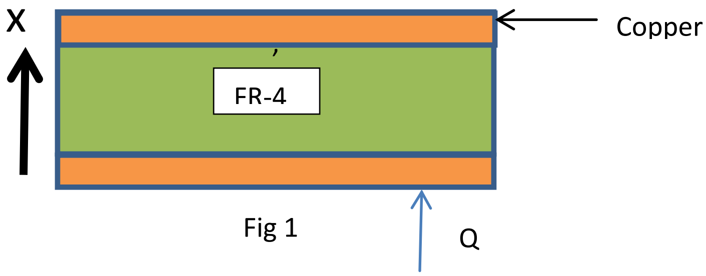


Fig 1

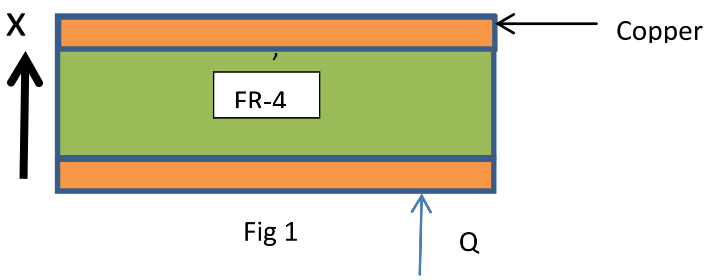


Fig 1

$$\frac{\partial T}{\partial t} = \frac{1}{k^2}\frac{\partial^2 T}{\partial \beta^2} \qquad (3)$$

A closed form solution exists for (1) modified by the above transformation, but it involves several infinite series, which defy extraction. A better course of action is to use a numerical solution. For the finite difference approximation to (3), the time derivative will be replace by a forward difference and the geometric derivative by a centered difference. The result is

$$T_i^{n+1} = T_i^n + \left(\frac{1}{k^2}\frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\Delta\beta^2}\right)\Delta t \qquad (4)$$

It is shown in Reference 1 that a numerical solution for (4) will converge provided

$$\Delta t \le \frac{k}{2}(\Delta\beta)^2 \qquad (5)$$

**<u>Results</u>**

A numerical integration of Equations 4 and 5 will now be examined for temperature ramps of 2, 4, and 6 degrees Fahrenheit/second. Initially, the board is at a uniform temperature of $300^0$F. The temperature ramp is completed when the surface temperature of the copper reaches $600^0$F. The temperature ramps are shown in Figure2.

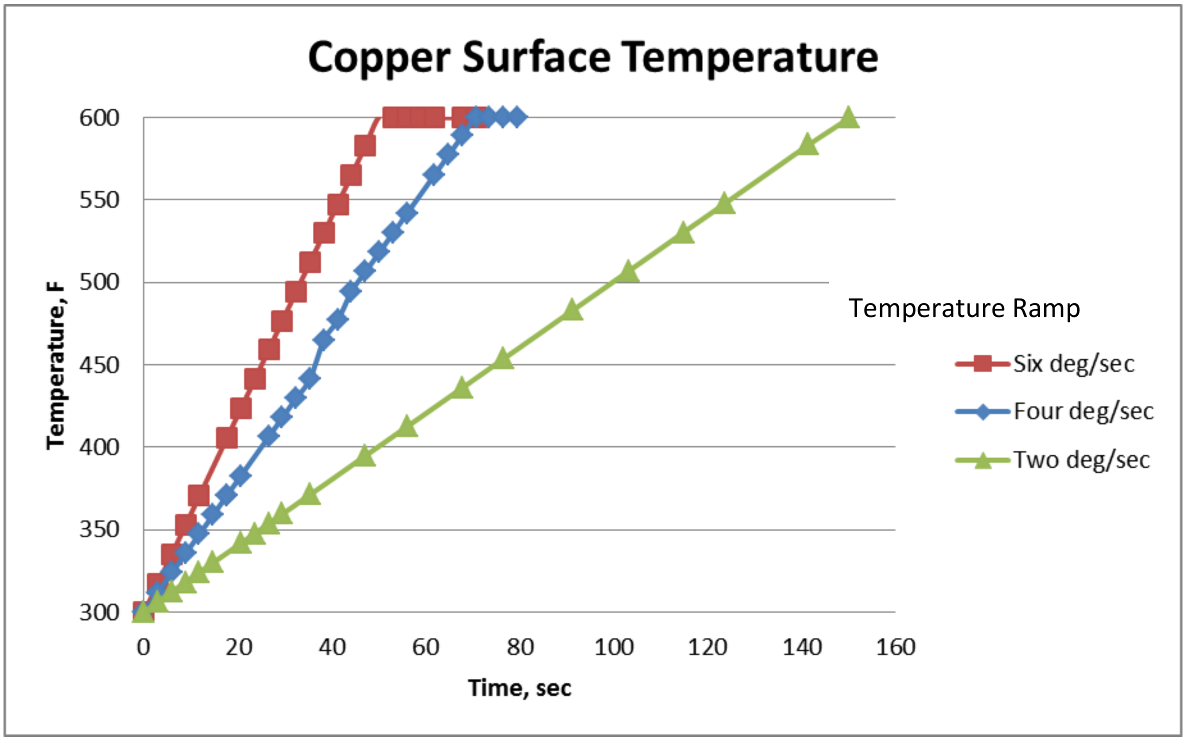


Figure 2

The integration of (3 and 4) also defines the temperature within the board as it passes along the temperature ramp. Figure 3 shows the board's temperature profile when the surface of the board first reaches $600^0$F.

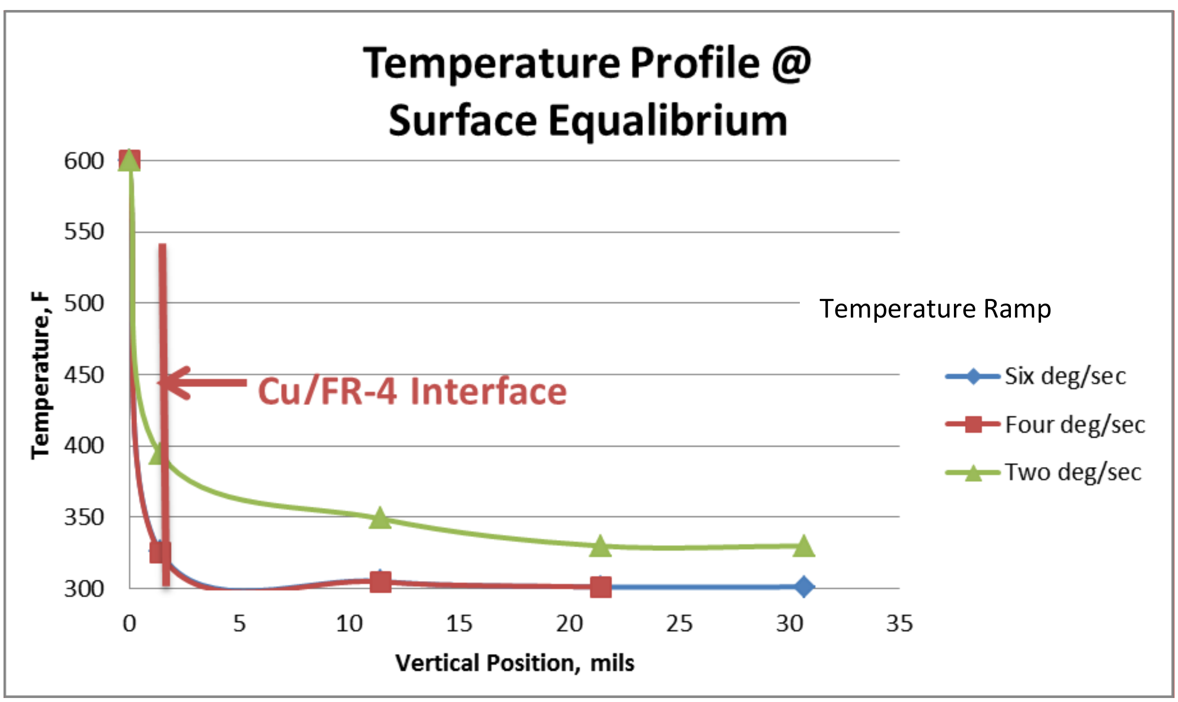


Figure 3

As seen, the temperature gradient is large in the outer portion of the board. Then, beyond the copper/FR-4 interface, the gradient rapidly approaches zero. This is caused by the favorable heat transfer properties of copper and the adverse heat transfer properties of FR-4. As seen later, this abrupt behavior causes large shear stresses to form at the interface, which can result in copper delamination.

In order to estimate the shear stress it is first necessary to calculate the difference in the temperatures of the copper and the substrate. For this purpose, the characteristic temperature of the PCB component is defined as the average temperature of each component. The result is presented in Figure 4 which shows the temperature difference as a function of temperature ramp. The temperature differential is obviously nonlinear. The rate of change in the temperature differential rapidly changes as the temperature ramp increases. It then becomes nearly constant at the high end.

In Reference (2), it is shown that the shear stress at the interface is

$$\tau = \frac{1}{L}\frac{E_{cu}E_L t_{cu} t_L}{E_{cu}t_{cu} + E_L t_L}(\alpha_{cu} - \alpha_L)\Delta T \qquad (6)$$

Where $\alpha$ is the coefficient of thermal expansion, L is the characteristic length of the copper feature and $\Delta T$ is the temperature differential. The shear stress is presented as a function of the copper feature size and ramp rate in Figure 5. It will be noticed that the shear stress increases rapidly as the feature size diminishes. In fact, one can expect to experience shear stresses in excess of 30K psi for present day PCBs.

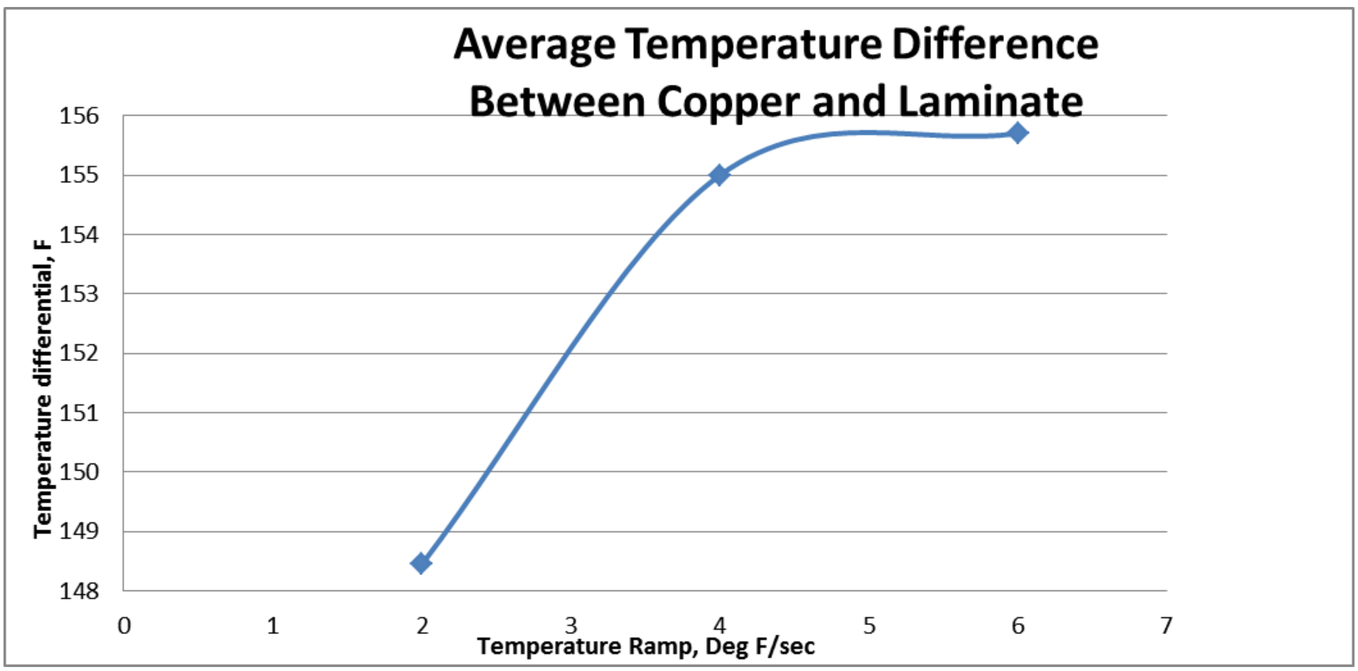


Figure 4

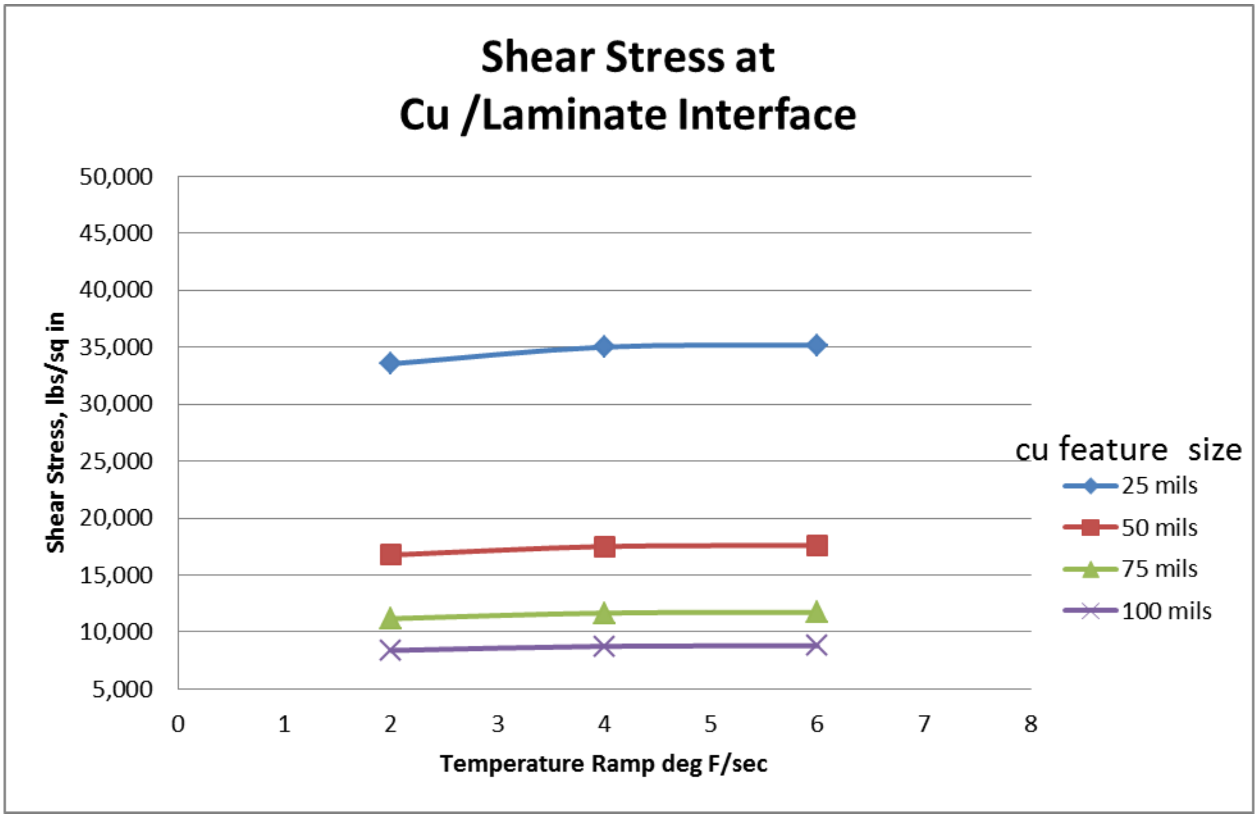


Figure 5

## SUMMARY

The analysis above has developed a technique for numerically integrating the one dimensional time dependent Fourier heat conduction equation for a PCB which is stressed by a heating ramp. This in turn, will quantify the relationship between the process and design parameters and the stresses incurred during a thermal excursion. The analysis shows there are substantial temperature gradients at the interface of the copper and FR-4 components. It is furthermore shown, these temperature gradients can produce very large shear stress at the copper/FR-4 interface. This stress is inversely proportional to the size of the copper feature and directly proportional to the gradient of the heating ramp.

## REFERENCES

(1) Jakob , Max Heat Transfer, John Wiley and Sons, New York, 1962
(2) J. Lee Parker, Proceedings of IPC Apex, April 2011

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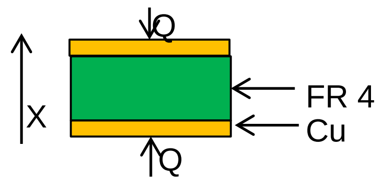
# Agenda

- Problem statement
- Develop analytical model
- Determine time history of board’s interior for different exterior temperature ramps
- Analysis

# Problem Statement

- Determine PCB temperature profile while passing through a temperature ramp $300^0$F--$600^0$F at variable heating rates




- Board model
  - Two sided board
  - Cu thickness 1.7 mils
  - FR-4 thickness 58 mils
- Board temperature is symmetrical
- Issue
  - Measuring board’s interior temperature (thermo-couples) will alter the local temperature
  - Solution: use a mathematical model

# Heat Transfer Analog

- The temperature profile is governed by Fourier's law of heat conduction which is in this case
- $\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$
- T is temperature
- $k = (c\gamma\mu)^{-1}$
- c is the specific heat
- ɣ is the density
- μ is the heat conduction per unit area and time

# Boundary Conditions

- The temperature at the surface of the copper is equal to the ramp temperature
- The temperature gradient in the board’s center is zero
- The temperature of the entire board at t=0 is $300^{0}\mathrm{F}$

# Additional Requirements

- For the solution to be unique $\frac{\partial T}{\partial x}$ must be continuous
- This requirement is violated at the Cu/FR-4 interface
- The transformation x=kξ will relieve the issue
- $\frac{\partial T}{\partial t} = \frac{1}{kc\gamma}\frac{\partial^2 T}{\partial \xi^2}$

# Observations

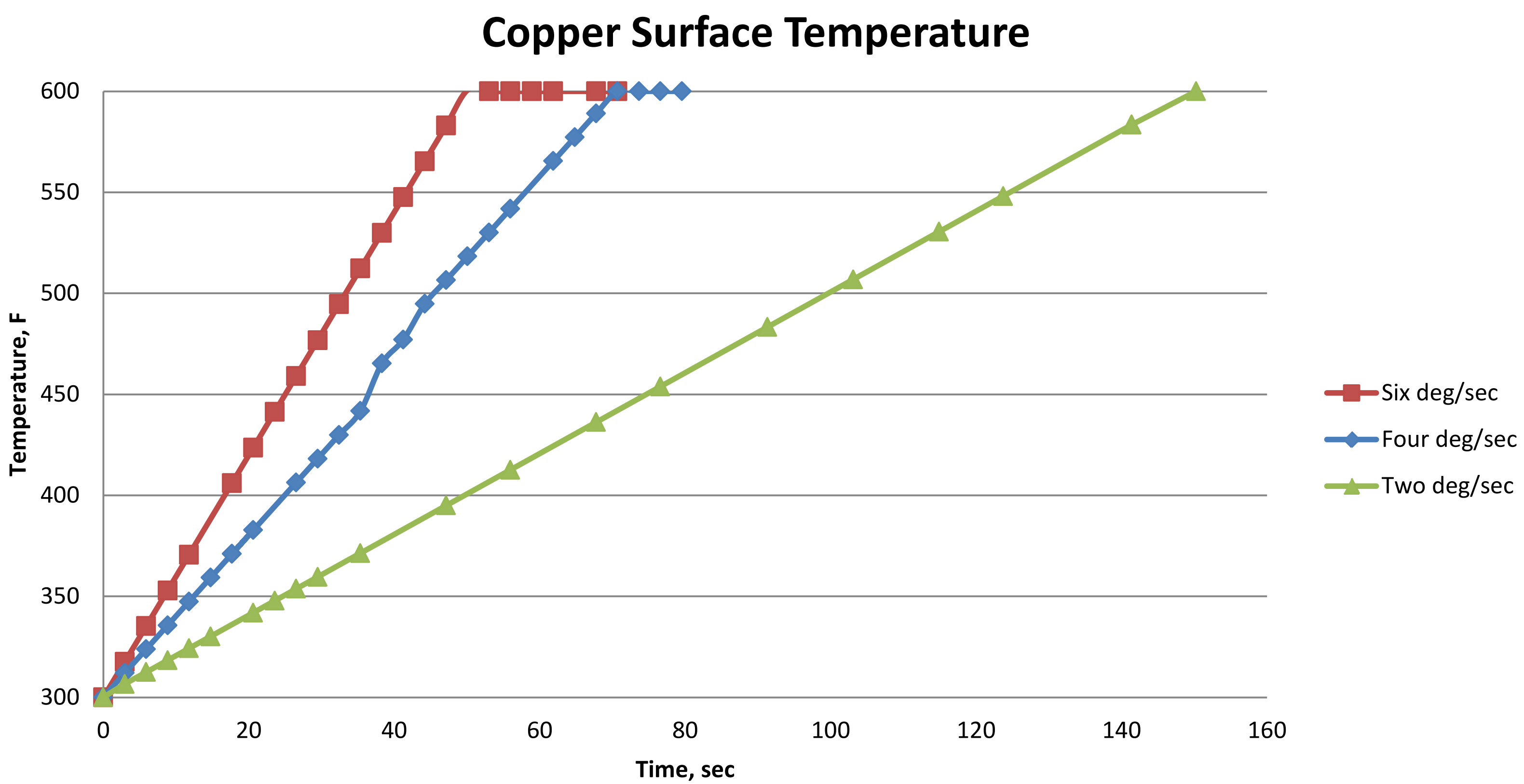
- The constant $\frac{1}{k c \gamma}$ changes at the interface
- Consequently a closed form solution is not practical
- The alternative is a finite difference integration

# Finite Difference Approximation

- Approximations
  - Time derivative approximation, forward finite difference
  - Geometrical derivative approximation, centered difference
- Finite difference approximation for Fourier's Law
- $(T_j)^{n+1} = T_j{}^n + \frac{1}{ck\gamma}\left[\frac{(T_{j+1} - 2T_j + T_{j-1})^n}{\Delta\xi^2}\right]\Delta t$
- The solution will converge provided
  - $\Delta t < \frac{c\gamma}{2}\Delta\xi^2$

# Analysis

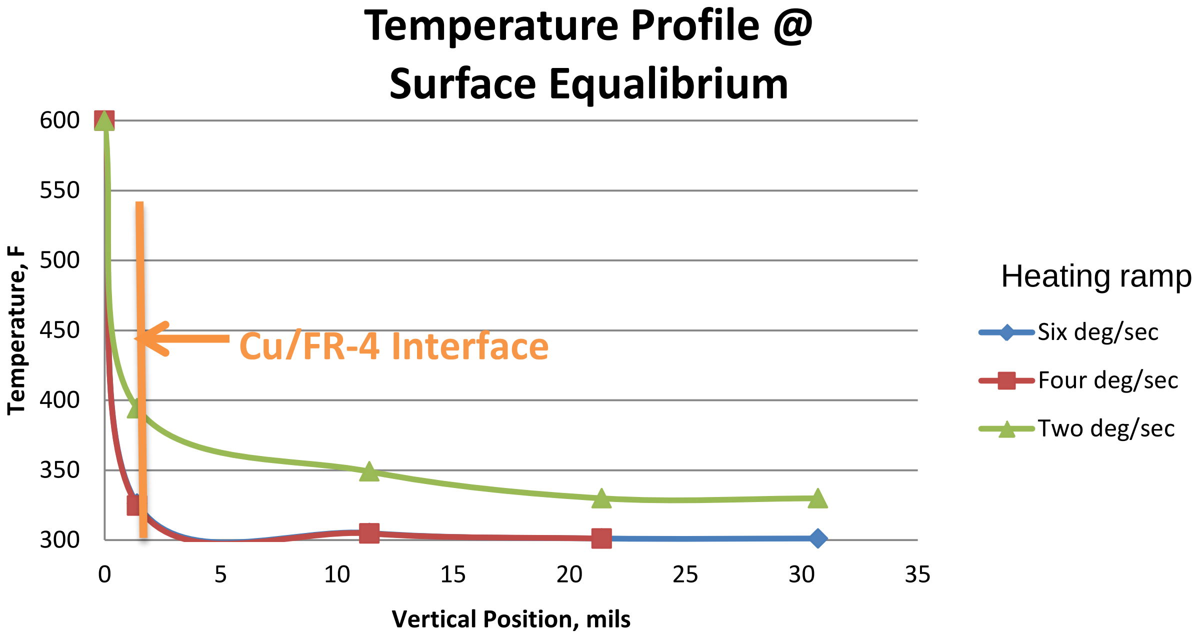
- Objective: estimate the maximum shear stress between Cu and FR-4
- Approach:
  - Integrate the heat transfer equation using several temperature ramps
  - Define the initial temperature as $300^{0}$F throughout board (simulating pre-heat)
  - After ramp temperature reach's $600^{0}$F, the surface of the board remains at $600^{0}$F



Copper Surface Temperature
Temperature, F
600
550
500
450
400
350
300
0
20
40
60
80
100
120
140
160
Time, sec
Six deg/sec
Four deg/sec
Two deg/sec

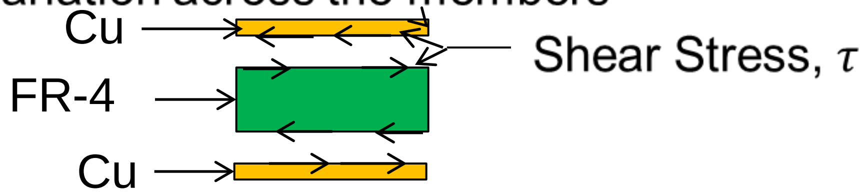
# Temperature Profile

- Between the Cu surface and the FR-4 interface the temperature gradient is large
- At the interface the temperature gradient falls and approaches zero



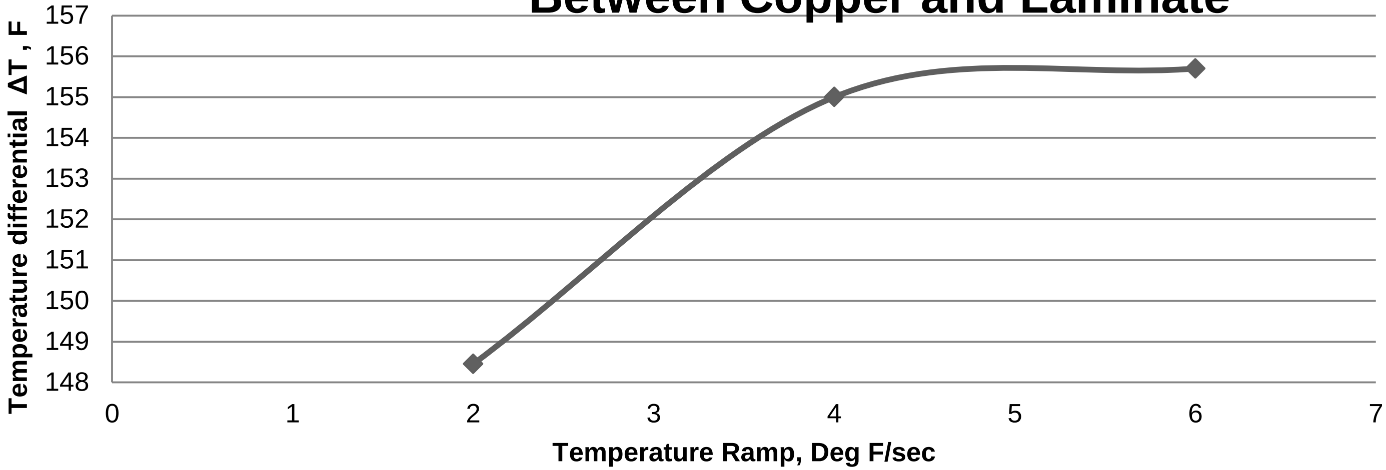
# Stress Analysis

- The shear stress at the Cu/FR-4 interface is proportional to the temperature variation across the members



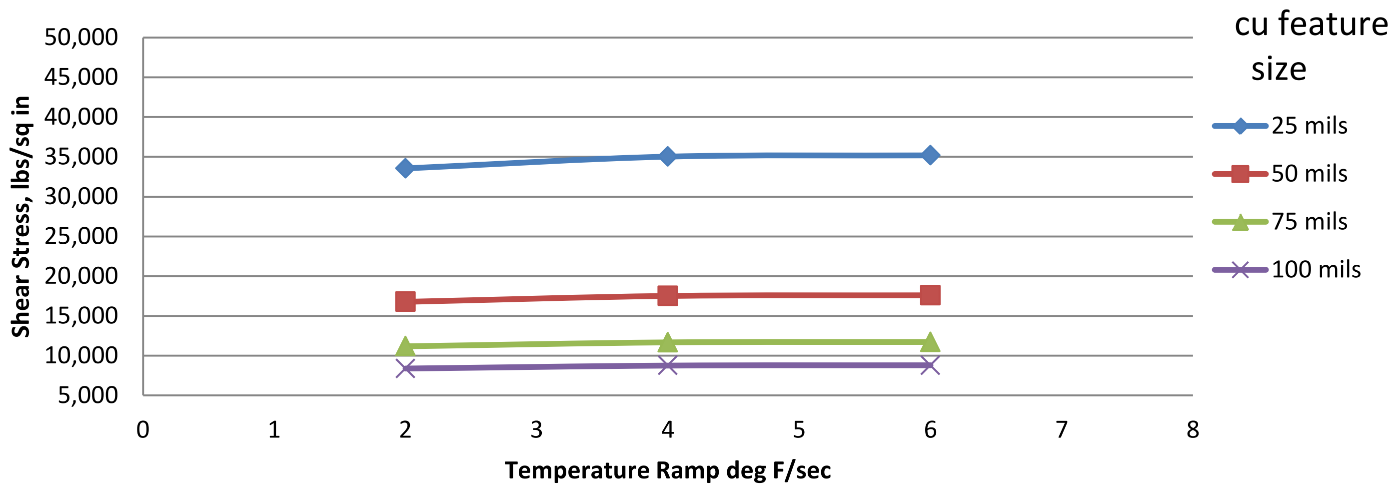

- The affective temperature $\Delta T_a$ is defined as the difference of the average temperatures across each member, $T_{cu}$ and $T_{la}$
- The shear stress is
- $\tau = \frac{1}{L}\left[\frac{E_{cu}\,E_{la}\;t_{cu}t_{la}}{E_{cu}t_{cu}+E_{la}t_{la}}\right](\propto_{la} - \propto_{cu})\Delta T_a$
- E modulus of elasticity
- t thickness
- L characteristic length of Cu feature
- $\propto$ coefficient of thermal expansion
- $\tau$ the shear stress at the interface
- It is noted that the shear stress is inversely proportional to the size, L, of the Cu feature

**Average Temperature Difference Between Copper and Laminate**




- When the temperature ramp is large the differential temperature is insensitive
- When the temperature ramp is <5 deg/sec the differential temperature changes rapidly with temperature ramp

# Shear Stress at Cu /Laminate Interface




**Observations:**

**Shear stress is a near linear function of the temperature ramp speed**

**Shear stress is a non-linear function of the Cu feature size**

**At or below 25 mil feature sizes, shear stress becomes robust and sensitive to size**

# Summary

- Fourier’s time dependent heat transfer equation can be solved numerically provided the appropriate restrictions are meet
- When a PCB experiences a thermal excursion one can anticipate:
  - Large temperature gradients in the board
  - An appreciable shear stress can develop between the copper outer layer and the FR-4 laminate
  - The shear stress between the outer layer copper feature and the laminate is inversely proportional to the feature size and sensitive to variations in size (printing and etching)

*Thank you*